frequency  $\gamma_{\alpha}$  is made smaller, narrowing the lineshape. We see in Fig. 2(b) that at even higher pump intensities,  $\gamma_{\alpha}$  continues to shrink, passing through zero, and the imaginary part of  $\alpha_{yy}^{ee}$  changes sign, indicating gain. As the pump intensity continues to increase,  $\gamma_{\alpha}$  continues to grow more negative and the lineshape begins to broaden.

Even though we have gain at the pump intensities in Fig. 5(b), we still do not have lasing because the gain is not large enough to overcome radiative losses.

## 4. Conclusion

We have presented a finite element method simulation for a microphotonic lasing system. We have shown how to achieve a massive speedup in the simulation by separating the various fields into fields that oscillate at the carrier frequencies  $\omega_1$  or  $\omega_2$ , with slowly changing complex valued amplitudes. A demonstration of this simulation was provided with a two dimensional model of a one dimensional cylindrical spaser array as an example. The threshold pump intensity for this array was determined. Finally, we have shown how the linewidth of the lasing transition changes for various pump intensities.

## Acknowledgments

Chris Fietz would like to acknowledge support from the IC Postdoctoral Research Fellowship Program. Work at Ames Laboratory was supported by the Department of Energy (Basic Energy Science, Division of Materials Sciences and Engineering) under contract no. DE-ACD2-07CH11358.