Electromagnetic behaviour of left-handed materials

M. Kafesaki\textsuperscript{a,e,*}, Th. Koschny\textsuperscript{a,b}, J. Zhou\textsuperscript{b}, N. Katsarakis\textsuperscript{a,c}, I. Tsiapa\textsuperscript{a}, E.N. Economou\textsuperscript{a,d}, C.M. Soukoulis\textsuperscript{a,b,e}

\textsuperscript{a}Foundation for Research and Technology, Hellas (FORTH), Institute of Electronic Structure and Laser (IESL), P.O. Box 1527, 71110 Heraklion, Crete, Greece
\textsuperscript{b}Ames Laboratory & Department of Physics and Astronomy, Iowa State University, Ames, IA, USA
\textsuperscript{c}Science Department, Technological Educational Institute of Crete, Heraklion, Greece
\textsuperscript{d}Department of Physics, University of Crete, Heraklion, Crete, Greece
\textsuperscript{e}Department of Materials Science and Technology, University of Crete, Heraklion, Crete, Greece

Abstract

Using periodic materials composed of split-ring resonators (SRRs) and continuous wires is, up to now, the most common way of obtaining left-handed behaviour in the microwaves regime. In this work, using transmission simulations and measurements, we examine the electromagnetic behaviour of those materials, focusing mainly on their response to an external electric field and its crucial role in the achievement of left-handed behaviour. Moreover, we examine and theoretically demonstrate the possibility of the SRRs
design to give left-handed behaviour in the infrared and optical regime, as well as the possibility of obtaining left-handed behaviour using an alternative for the SRRs
design composed of pairs of short wires.

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1. Introduction

In recent years, much interest has been shown on a new class of materials, the so-called left-handed materials (LHMs) or negative index materials (NIMs). LHMs are artificial composite materials which, over some finite-frequency range, exhibit a negative index of refraction, as a result of simultaneously negative effective electrical permittivity ($\varepsilon$) and magnetic permeability ($\mu$). LHMs were discussed first by Veselago in 1967 [1], and he predicted very unusual physical properties for those materials, like opposite phase and group velocities, reversal of Snell’s law (negative refraction) and of Doppler shift, etc. Despite those exciting and unusual properties that can give LHMs a unique power in the control of electromagnetic (EM) waves, no practical implementation of such materials occurred until recently, mainly due to the lack of $\mu<0$ materials in technologically interesting frequency regions. The first practical implementation of LHMs came only in 2000 [2], by Smith et al., following ideas by Pendry et al. [3]; this first left-handed material was a periodic combination of metallic rings with gaps, known as split-ring resonators (SRRs), and continuous wires. SRRs, under the influence of an external alternating magnetic field, behave like inductor–capacitor (LC) circuits [3], exhibiting a magnetic resonance (corresponding to resonant oscillation of circular currents in the rings), followed by a negative permeability regime. The frequency of this magnetic resonance, $\omega_m = 1/\sqrt{(LC)}$, is a function of geometrical SRR parameters. The continuous wires, on the other hand, behave as a free electron plasma with a reduced plasma frequency, $\omega_p$, which depends also on the geometrical parameters of the system [3].

Since the first demonstration of a LHM, several LH structures were fabricated and tested, most of them
combinations of SRRs and wires, operating in the microwave regime. An intensive effort to understand their physical properties, to optimize them, to extend their frequency of operation, and to identify all their potential applications was carried out [4,5].

In this study, we describe some of our recent efforts to understand the behaviour of the SRRs&wires materials, to obtain LHM s operating in the IR and optical range and to devise alternative to the SRRs&wires designs for achieving optimized LHM s. The study is organized as follows: In Section 2, we examine the effect of the electric response of SRRs on the spectrum of LHM s; taking into account this effect, we propose a criterion to unambiguously identify the left-handed regimes in SRRs&wires transmission spectra. In Section 3, we examine the role of the SRR symmetry on the behaviour of SRR and on its capability to create left-handed materials. We show that effects associated with SRR asymmetries can be used to trace the magnetic resonance of micrometre scale SRR structures and to demonstrate the existence of a magnetic response in the THz regime. In Section 4, we discuss the possibility of achieving optical SRR-based left-handed materials and the frequency limitations of the left-handed behaviour. Finally, in Section 5 we examine an alternative to the SRRs approach for achieving negative magnetic permeability and negative index of refraction, namely pairs of short wires or strips.

The results presented here are mostly transmission (T) simulations and measurements. The simulations have been performed using the finite integration technique, employed through the Microwave Studio commercial software; moreover, a retrieval procedure (approximating the material with a homogeneous effective medium) [6] has been employed to extract the effective parameters (ε, μ, n) of the studied systems from transmission and reflection data, and to unambiguously prove the negative index behaviour of those systems. The experimental results are free space transmission measurements, using a standard network analyzer system appropriate for microwave characterization.

2. Electric response of SRR systems and its role in the electric response of left-handed materials

Apart from the magnetic response of the SRRs [3] which is responsible for their capability to create LHM s, SRRs, as every finite metallic system, present also a resonant electric response. This resonant electric response, which can be described with a Lorentz-type effective permittivity, corresponds to strong parallel currents at the sides of the SRR which are parallel to the external electric field (E). Since the frequency of this response (f_m) is usually larger than the magnetic resonance frequency, f_m = ω_m / 2π, of the SRR, the electric response of the SRRs was not initially taken into account in the study of LHM s; i.e., to determine the negative ε regimes of SRRs&wires materials only the wires were employed.

However, later detailed theoretical and numerical studies [7] showed that the electric SRR response, due to its resonant nature, is in many cases crucial in the determination of the negative effective ε regimes of an SRRs&wires medium and thus for the correct identification of the left-handed regimes. Another important finding was that this electric SRR response is entirely preserved if one closes the SRR gaps, destroying thus the magnetic SRR.

Fig. 1. (a) Simulated (solid line) and measured (dashed line) transmission amplitude (S21) of a 5-unit cells (uc) along propagation direction periodic arrangement of SRRs. The dotted line shows the simulated S21 after closing the gaps of the SRRs. Notice that by closing the SRR gaps the dip at ~3 GHz disappears while the rest of the spectrum remains unaffected. This indicates that the ~3 GHz dip is magnetic in origin, while the dip after 8 GHz corresponds to negative ε behaviour. (b) Simulated (solid line) and measured (dashed line) transmission (S21) spectra of a left-handed material composed of SRRs (like the ones described in panel (a)) and wires, in a periodic arrangement (5 uc along propagation direction). The dotted line shows the S21 of the combined material of closed SRR and wires (CCMM) and the dotted-dashed line the S21 for the wires only. Notice that the plasma frequency of the CCMM (at ~5 GHz) is much lower than the plasma frequency of wires only, which is at ~9 GHz. Notice also that the only difference between the transmission of LHM and CCMM is the left-handed peak at ~3 GHz, showing that the CCMM carries all the electric response of the LHM. The geometrical parameters of the system are: uc size a_x × a_y × a_z = 8.8 × 8.8 × 6.5 mm^3, l = 7 mm, d = w = 0.2 mm, t = 0.9 mm (see inset), metal depth = 30 μm, wires width = 0.9 mm. The SRRs and wires are printed on opposite sides of a PCB board of thickness 1.6 mm and ε = 2.8. The wires are placed symmetrically to the SRRs, along the imaginary line connecting the two SRR gaps.
response (this holds for SRRs symmetric relative to the external \(E\)).

Therefore, the closing of the SRR gaps can lead to the identification of both the negative \(\mu\) and the negative \(\varepsilon\) regimes of a SRRs system, as is shown in Fig. 1(a), where the transmission for a SRRs and a closed-SRRs (CSRRs) system is presented. Combining the closed-SRRs with wires, one can demonstrate the total electric response (effective \(\varepsilon\)) of the combined medium. This electric response, as shown in Fig. 1(b), is very different from that of the wires taken separately; the most important difference for the LHM creation and understanding is the large downwards shift of the effective plasma frequency compared to that of the wires only, \(f_p = \omega_p/2\pi\). This shift poses stringent requirements for the achievement of LH behaviour: for a SRRs&wires medium to be left handed, the magnetic SRR resonance frequency \(f_m\) should lie not only below \(f_p\) but also below the new cut-off frequency, \(f_{p}'\). (Note that if \(f_p < f_m < f_{p}'\), a case very common in practical implementations, ignoring the SRR electric response and its effect has as a result wrong identification of left-handed peaks.)

The above-mentioned results lead to an easy and correct way to identify both the negative \(\mu\) and the negative \(\varepsilon\) regimes in SRRs&wires systems (in the long-wavelength limit), through simple transmission experiments or simulations: The gaps in the CSRRs&wires transmission spectra show the true negative \(\varepsilon\) regimes of an SRRs&wires system, while the different features between CSRRs&wires and SRRs&wires transmission are exclusively due to the negative \(\mu\) of the SRRs. If those features correspond to a peak, then this peak is left handed, as the one in Fig. 1(b), at \(\sim 3\) GHz.

Transmission simulations like the ones presented in Fig. 1, as well as associated retrieval procedures for the determination of the effective \(\varepsilon\) of the systems, indicate that the effective \(\varepsilon\) of SRRs&wires systems has the form of a sum of a plasmonic and a Lorenz-type permeability, i.e. it can be obtained by adding the effective \(\varepsilon\) of the SRRs to the effective \(\varepsilon\) of the wires [7].

3. Electric field-induced excitation of the magnetic SRR resonance and its exploitation

Investigating the effect of the SRR orientation on the left-handed behaviour of a SRRs&wires system, for various SRR types, we found that another aspect of the electromagnetic behaviour of the SRR crucial for its ability to create LHMs is the possibility of excitation of its magnetic resonance (i.e. the resonant oscillation of the circular currents around its rings) by the external electric field, \(E\). This electric field-induced excitation of the magnetic resonance (EEMR effect) occurs whenever the SRR is asymmetric with respect to \(E\), as is shown and explained in Fig. 2(b); it occurs even for incidence normal to the SRR plane, where the magnetic flux through the SRR is zero [8,9]. The effect of the EEMR on the effective electromagnetic response of an SRRs medium is the imposition of a strong electric resonance (resonance in the effective \(\varepsilon\)) at the magnetic resonance regime of the SRR. This resonant \(\varepsilon\), followed by a negative \(\varepsilon\) regime which manifests itself as a dip in the SRR transmission spectrum, see Fig. 2(a), is the result of a non-zero average polarization induced by the resonant circular currents.

In combined SRRs&wires systems aimed to create LHMs, the presence of this strong electric resonance (i.e. of large positive \(\varepsilon\)) in regimes where a negative \(\varepsilon\) response is required constitutes a serious impediment for the achievement of left-handed behaviour [8]. For creation of two-dimensional (2D) or three-dimensional (3D) left-handed materials this impediment can be avoided only by employing symmetric SRR designs e.g. multigap SRRs [10].

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**Fig. 2.** (a) Transmission amplitude, \(S_{21}\), for a single SRR with geometrical parameters described in Fig. 1, for the four possible orientations (see inset) of the SRR with respect to an external EM field. For the orientation (d) (normal incidence, i.e. no magnetic response possible, and SRR asymmetric in respect to \(E\)) the dip in the transmission is a manifestation of the negative \(\varepsilon\) response of the SRR due to the EEMR effect. (b) Simple drawing of the charge and current for a non-symmetric single-ring SRR in an external electric field, \(E\) (pointing upwards). The interior of the ring shows simulations data for the polarization current component \(J_{\perp}\) at a fixed time for normal incidence; white color indicates upwards current and black downwards current. The asymmetry of the SRR leads to different electric field induced charges at the two top and bottom corners of the SRR and thus to a potential difference which is compensated by a circular current.
The EEMR effect, although it constitutes an impediment for the achievement of left-handed behaviour, is associated with a significant advantage for the study of SRRs: it can offer an indirect way to locate the SRR magnetic resonance regime in transmission experiments, since it offers the possibility to trace the magnetic resonance using normal incidence. This is particularly useful in the study of micro- and nano-scale SRR structures, where one cannot fabricate multistacks of SRRs easily, as it is essential for regular transmission experiments employing an external \( H \) normal to the SRR plane. This advantage offered by the EEMR effect was exploited recently very extensively: It was used to demonstrate experimentally the occurrence of a magnetic SRR resonance in the 6 THz [11], 100 THz [12] and 1.5 \( \mu \)m [13] regimes. The existence of negative \( \mu \) at that magnetic resonance of those systems was confirmed by corresponding transmission simulations, followed by effective parameters retrieval; in some cases it was confirmed also by associated oblique incidence reflection experiments [14].

4. Negative permeability towards optical frequencies

The novel and unique properties of LHMs, entailing unique capabilities in the manipulation of EM waves, led to very strong efforts to extend the left-handed behaviour from the microwaves, where it was initially demonstrated, in the THz and optical regimes, where telecommunications and imaging applications can undergo revolutionary changes. Since most of the current realizations of left-handed materials involve metallic elements, and the left-handed behaviour is based on the perfect conductor behaviour of those elements, the question that naturally arises is “as one goes to higher and higher frequencies, where the metal deviates more and more from the perfect conductor picture, how will the negative effective \( \varepsilon \) and \( \mu \) response of left-handed materials be influenced? Is still possible to achieve left-handed behaviour in the optical regime employing the current SRRs & wires topology?” Seeking an answer to this question, we examine here the magnetic response of SRRs as one goes to 1R and optical frequencies, i.e. to \( \mu \)m and \( \mu \)m scale SRRs. In this study we employ single ring instead of double ring SRRs because of the simplicity of the design and the increased possibilities for practical implementation.

In Fig. 3(a) we show the magnetic SRR resonance frequency for SRRs with 1, 2, and 4 cuts, as a function of the unit cell size, \( a \), of the structure, for micro- and nano-scale structures, together with the associated designs (Fig. 3(b)). One has to note here that for mm-scale SRRs the magnetic resonance frequency \( \omega_{m}^{2} = 1/LC \) scales as the inverse of the structure size, as both approximate analytical models and accurate numerical simulations reveal. As one can see from Fig. 3(a) though, at higher frequencies the linear scaling, \( \omega_{m} \propto 1/a \), of the magnetic SRR resonance breaks down, and the magnetic resonance frequency tends to a saturation value different for each type of SRR and higher for the four-cut SRR design (for larger cuts numbers

Fig. 3. (a) The scaling of the simulated magnetic resonance frequency, \( f_{m} = \omega_{m} / 2\pi \), as a function of the linear size, \( a \), of the unit cell for the one-, two-, and four-cut SRR designs shown in panel (b). Up to the lower THz region the scaling is linear. The maximum attainable frequency is strongly enhanced with the number of cuts in the SRR ring. The hollow symbols, as well as the vertical line at \( 1/a = 17.9 \mu \)m\(^{-1} \) indicate that no \( \mu < 0 \) is reached anymore. For the SRR designs employed the unit cell has the dimensions \( a \times a \) in the SRR plane and 0.614\( a \) perpendicular to it. The SRR is made of aluminum, simulated using a Drude-model permittivity. (c): Simulated \( \text{Re}[\mu_{eff}(\omega)] \) for a single-ring 4-cut SRR, for unit cell size \( a = 70, 56, 49 \), and 35 \( \mu \)m (left to right). The corresponding \( \text{Im}[\mu_{eff}(\omega)] \) curves (not shown here) show a peak of Lorenzian shape at the magnetic resonance. The half-width at the half-maximum of \( \text{Im}[\mu(\omega)] \) increases as the frequency increases. This shows that the losses are getting larger for higher frequencies.
the saturation value may be even higher—however, the picture of a homogeneous-like metamaterial, described by a uniform $\mu$, will be completely wrong). This saturation is accompanied by a reduction in the amplitude of the resonant permeability (see Fig. 3(c)), which ultimately ceases to reach negative values, due mainly to the increase of the losses.

As explained in detail in Ref. [15], the origin of the above mentioned saturation is the fact that the kinetic energy of current-carrying electrons of the SRR, $E_k$, becomes comparable to the magnetic energy, $E_m = L_I^2/2$ ($E_m$ decreases with the SRR size). Assigning to the kinetic energy an equivalent inductance, $E_k = (n_e V) m_e v_e^2/2 = L_e L_I^2/2$ ($n_e$ is the electrons number density, $V$ the effective (current-carrying) volume of the SRR, proportional to the actual volume, $m_e$ the electron mass and $v_e = I/S e_n$, the electron drift velocity, with $S$ the effective cross-section of the SRR ring—proportional to the actual cross-section). Using the above formulas the kinetic inductance can be expressed through the geometrical characteristics of the SRR [5,15]: taking into account a uniform SRR scaling, i.e. all lengths scaling proportional to the unit cell size, $a$, one can find that $L_e$ scales inversely with $a$, in contrast to the magnetic inductance, $L$, which scales proportionally to $a$; the capacitance also scales proportionally to $a$ and thus the magnetic resonance frequency has the following size dependence:

$$\omega_m = \frac{1}{\sqrt{(L + L_e)C}} \propto \frac{1}{\sqrt{\text{size}^2} + \text{const}}.$$  

The above formula explains the observed saturation of $\omega_m$ when going to nm scale structures. For millimetre-scale and larger SRRs, simple analytical calculations show that $L$ is much larger than $L_e$, leading to the linear scaling of $\omega_m$. Approaching the nm scale though, $L_e$ becomes dominant, leading to the $\omega_m$ saturation. The saturation value can be estimated by considering the above calculated $L_e$ and approximate formulas for the capacitance [5].

From the above simulations it can also be seen that the four-cut SRR design is superior to one- or two-cut designs, since it leads to the highest magnetic resonance frequency. It gives $\mu < 0$ behaviour for frequencies up to 500 THz, i.e. in the optical regime, demonstrating thus the possibility for optical left-handed materials using SRRs and wires.

5. Negative index materials using pairs of short wires

Most of the NIM implementations to-date employ the SRRs&wires topology proposed by Pendry. However, this topology presents certain inefficiencies, especially for applications in the THz regime and upwards: to demonstrate left-handed behaviour of SRRs and wires one has to employ a magnetic field perpendicular to the SRRs’ plane and thus propagation parallel to this plane; this requires multilayer samples, which is difficult with the current technological possibilities and impractical in terms of applications.

An alternative to the SRR design that seems to overcome the above mentioned inefficiencies presented by the SRR and is very promising for achievement and demonstration of left-handed behaviour in the THz and optical regime is a simplified design composed of a pair of short wires or short strips [16], as shown in Fig. 4. The short-wires pair ( unlike a single short wire) can behave like an SRR, exhibiting a resonant magnetic mode associated with resonant antiparallel currents in the two wires of the pair—see Fig. 4(a). This mode can be excited by an incident EM field with $k$ perpendicular to the layer where such pairs are printed—shown also in Fig. 4(a), allowing thus demonstration of negative $\mu$ behaviour with only one layer of short-strip pairs.

Apart of the resonant magnetic mode described above, short pairs present also a symmetric mode, corresponding to parallel currents in the wires of the pair and associated with a resonant electric response followed by a negative $\varepsilon$ regime (see Fig. 4(b)).

The existence of both a magnetic and an electric resonance in the pair led initially to the attempts to tune these two resonances properly, obtaining thus left-handed

![Fig. 4. The currents at the magnetic (panel (a)) and at the electric (panel (b)) resonant mode of a pair of short wires or slabs. Such currents can be exited by an external electromagnetic field like the one shown in the figure. Panel (c) shows the frequency of the magnetic and the electric resonances of the pair as one increases the length of the wires to approach the corresponding unit cell size, $a_e$. The deviation from the linear scaling $f_m \propto 1/l$, valid for an isolated pair, is due to the extra capacitance in the region between neighbouring unit cells along the external $E$; this region is marked with $\text{1}$ in panel (d).](image-url)
behaviour employing only short pairs. Detailed examination of the design, however, showed that in an isolated short pair the geometrical parameter that predominantly determines the frequency of both the electric and the magnetic resonance is the length of the wires; both frequencies scale as $1/\ell$ [17]. This indicates the nonfeasibility of a separate tuning of those resonances and thus of the achievement of left-handed behaviour employing isolated short pairs.

However, tuning both resonances separately seems to become possible though interaction of pairs belonging to neighbouring unit cells along the electric field direction. The additional inter-pair capacitance acquired if neighbouring pairs come close (“connected” in parallel with the inter-wires capacitance in the pair) affects differently the magnetic and the electric resonance frequency and can lead to the possibility of a separate tuning of those frequencies and thus to their coincidence. Modifications of the design of the two wires, as to increase this capacitance [18], give additional possibilities for the achievement of left-handed behaviour using only pairs of short wires.

Another, more convenient and safe approach to obtain LH behaviour from short pairs is to combine them with continuous wires, which provide a large negative $\varepsilon$ band due to their plasmonic behaviour, and thus to exploit only the negative $\mu$ behaviour of the pair. This approach has been exploited recently for the demonstration of left-handed behaviour in both GHz and THz regimes [19,20].

6. Conclusions

Using transmission simulations and measurements, we studied the electromagnetic behaviour of composite materials of SRRs and wires. We focused mainly on the response of the SRRs to an external electric field, $E$, and its influence in the achievement of left-handed behaviour in a SRRs&wires system. We found two dominant types of electric response in the long-wavelength limit: (a) An electric short-wire-like response, which is responsible for a significant downward shift of the effective plasma frequency of an SRRs&wire structure compared to the plasma frequency of wires only. (b) An $E$-induced excitation of the magnetic SRR resonance, which occurs when the SRR is asymmetric with respect to $E$, and constitutes an impediment for the achievement of left-handed behaviour. Apart from the electric response of the SRRs, we also examined their possibility to give negative magnetic permeability in the infrared and optical regime. We found negative permeability up to $500\,\text{THz}$ in a simple single-ring four-cut SRR design. In even higher frequencies, the increased kinetic inductance of electrons (compared to the magnetic inductance), together with the losses seem to kill the negative magnetic SRR response. Finally, we studied the possibility to obtain left-handed behaviour using pairs of short wires instead of SRRs. We found that the short-pair design can replace very successfully the SRR in achieving negative permeability; moreover, under certain conditions, it can also produce left-handed behaviour itself, without the requirement of additional continuous wires, while it presents certain advantages in terms of practical implementation. The short-pair design, because of its simplicity and the ease in the characterization, constitutes a promising approach for the achievement of left-handed behaviour at THz and optical frequencies.

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