Negative-Index Materials: New Frontiers in Optics**

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A lot of recent interest has been focused on a new class of materials, the so-called left-handed materials (LHMs) or negative-index materials, which exhibit highly unusual electromagnetic properties and hold promise for new device applications. These materials do not exist in nature and can only be fabricated artificially; for this reason, they are called metamaterials. Their unique properties are not determined by the fundamental physical properties of their constituents, but rather by the shape and distribution of the specific patterns included in them. Metamaterials can be designed to exhibit both electric and magnetic resonances that can be separately tuned to occur in frequency bands from megahertz to terahertz frequencies, and hopefully to the visible region of the electromagnetic spectrum. This article presents a short history of the field, describes the underlying physics, and reviews the experimental and theoretical status of the field at present. Many interesting questions on how to fabricate more isotropic LHMs, on how to push the operational frequency to optical wavelengths, how to reduce the losses, and how to incorporate active or nonlinear materials in LHMs remain to be explored further.

1. Introduction

1.1. History of Negative-Index Materials (or Left-Handed Materials)

Electromagnetic metamaterials are artificially structured media with unique and distinct properties that are not observed in naturally occurring materials. More than three decades ago, Victor Veselago[1] predicted many unusual properties of a hypothetical (at that time) isotropic medium with simultaneously negative electrical permittivity ($\varepsilon$) and magnetic permeability ($\mu$), which he named a left-handed material (LHM). As Veselago showed, LHMs display unique “reversed” electromagnetic (EM) properties as a result of an EM wave in such a medium having the triad $\mathbf{k}$ (wave vector), $\mathbf{E}$ (electric field), and $\mathbf{H}$ (magnetic field) left handed and, hence, exhibiting phase and energy velocities of opposite directions. It follows also that a LHM is characterized by a negative refractive index ($n$); hence, their alternative name, negative-index materials (NIMs). The latter leads, in particular, to Cherenkov radiation, a Doppler shift, radiation pressure, and even Snell’s law being reversed in LHMs.
properties open up a new regime in physics and technology (e.g., almost zero reflectivity at any angle of incidence), provided that such LHMs can be realized.

Veselago’s initial suggestion remained completely hypothetical, as naturally occurring materials do not provide such properties, until a significant breakthrough was announced in 2000: Smith and co-workers\[2,3\] presented evidence for a composite medium—interlaced lattices of conducting rings and wires (Fig. 1a)—displaying negative values of \( \varepsilon \) and \( \mu \).

For the development of this first LHM, Smith and his colleagues followed the pioneering work of Pendry et al., who in 1999 developed designs\[4\] for structures that are magnetically active, although made of non-magnetic materials. One of those structures is the so-called split-ring resonator (SRR) structure, composed of metallic rings with gaps (see Fig. 1a and b), which has been widely adopted as the model for creating negative \( \mu \) at gigahertz frequencies. The SRR structure has proven a remarkably efficient means of producing a magnetic response, and has been recently scaled down in size (and thus upwards in frequency) to produce metamaterials active at terahertz frequencies (see discussion below). Pendry and his colleagues have applied designs related to the SRR to magnetic resonance imaging studies, centered in the megahertz region of the spectrum.\[5\]

A key property of a LHM is that it possesses a negative index of refraction—a novel and remarkable material property. One of the most dramatic—and controversial—predictions of the LHMs was that by Pendry,\[6\] who stated that a thin negative-index film should behave as a “superlens”, providing image detail with a resolution beyond the diffraction limit, to
which all positive-index lenses are subject. Conventional positive-\( n \) lenses require curved surfaces to bend the rays emanating from an object to form an image. Yet, Pendry and, earlier, Veselago noted that negative-\( n \) lenses are not subject to the same constraint: they found that a planar slab of material with an \( n \) of –1 could also produce an image. For this lens, diverging rays from a nearby object are negatively refracted at the first surface of the slab, reversing their trajectory so as to converge at a focus within the material (Fig. 2). The rays diverge from this focus and are again negatively refracted at the second surface, finally converging to form a second image just outside the slab. Although it produces an image, the planar lens differs from conventional curved-surface lenses in that it does not focus parallel rays and has a magnification that is always unity.

Upon careful reexamination of this planar lens, Pendry found that it is, in principle, also capable of recovering the evanescent waves emanating from an object.\(^6\) (The EM field of an object includes not only propagating waves, but also near-field “evanescent” waves that decay exponentially as a function of the distance from the object.) These evanescent waves carry the finest details of the object, but their recovery by conventional positive-index lenses is minimal and only at the very near field, which leads to a resolution no better than roughly one-half of the illuminating wavelength—the diffraction limit. Pendry found that, in a planar negative-index lens, an evanescent wave decaying away from an object grows exponentially in the lens; on exiting the lens, the wave decays again until it reaches the image plane, where it has the same amplitude with which it started (Fig. 3).\(^6\) Unlike any other lens, the resolution limit of the planar negative-index lens is determined by how many evanescent waves from the object can be recovered, rather than by the diffraction limit (in practice, several stringent requirements limit perfect focusing). The radical idea of beating the diffraction limit led to many objections, initially suggesting that this was impossible.\(^7,8\)

In the last four years (for recent reviews of the LHM field, see Smith et al.\(^7\) and Ramakrishna\(^8\)) different groups, by careful experiments, sophisticated simulations, and physical analysis, have come up with new, optimized structures,\(^9–18\) mostly at microwave frequencies (see Fig. 4, where the best transmission left-handed (LH) peak is shown;\(^14\) losses are only –0.3 dB cm\(^{-1}\)). These structures exhibit an unambiguously negative index of refraction; therefore, the widespread acceptance of the existence of negative index of refraction materials (NIMs) is well established.

Recently, many groups have fabricated samples and observed\(^19–24\) negative \( \mu \) in the terahertz region. In Figure 5, we present results for SRRs that give a negative \( \mu \) at 100 THz.\(^20\) Enkrich et al. in Karlsruhe managed to fabricate\(^22\) SRRs that give a negative \( \mu \) at 200 THz! This is an amazing accomplishment for the LHM field. Thus, it is a great challenge of this field to be able to combine the SRR structures, which give
negative $\mu$, with the wires, which give negative $\varepsilon$, to produce a NIM at terahertz and optical frequencies. However, as one moves to optical frequencies, the losses of the metallic elements of the LHMs might be a problem and need to be addressed in detail.

Closing this subsection, we have to mention that the achievement of negative-index behavior by combining SRRs and wires (which may not be easy), even in the microwave regime, is by no means guaranteed; there is always a high probability (at least for the most commonly used designs) that the negative permeability regime will not fall in the negative permittivity regions but in the positive, or that unwanted resonances in $\varepsilon$ (resulting from asymmetries of the design) will hinder simultaneous $\varepsilon < 0$ and $\mu < 0$ behavior. To establish design rules for the achievement of negative-index behavior using SRRs and wires, one has to consider very carefully the behavior of both the metamaterial components (SRRs and wires) in the presence of an external EM field, as well as the interactions of those components (see, e.g., Kafesaki et al. and Koschny et al.\[26\]). In general, negative-index behavior is favored in SRR designs with high symmetry, combined with symmetricaly placed wires (to minimize the SRR–wire interaction).

### 1.2. Negative Refraction in Photonic Crystals

A different approach for achieving a negative index of refraction\[27–33\] is to use photonic crystals (PCs). PCs can be made from dielectrics only and can, in principle, have much smaller losses than the metallic LHMs, especially at high frequencies, and even in the optical range. In PCs, to achieve negative refraction the size and the periodicity of the “atoms” (the elementary units) should be of the order of the wavelength. In regular LHMs, the size of the unit cell is much smaller than the wavelength, which allows for the application of a uniform effective medium theory for the determination of effective $\varepsilon$ and $\mu$ values of the metamaterial. In PCs, no effective $\varepsilon$ and $\mu$ can be defined, although the phase and energy velocity can be opposite to each other as in regular LHMs.

Both negative refraction and superlensing have been demonstrated in PCs. Cubukcu et al. used dielectric rods to show negative refraction\[31\] as well as subwavelength focusing\[32\] in PCs at microwave frequencies. One of their results is shown in Figure 6, where one can see subwavelength resolution obtained from a PC consisting of a square array of dielectric rods in air (the rods’ dielectric constant is $\varepsilon = 9.61$, diameter $d = 3.15$ mm, length $l = 15$ cm, and lattice constant $a = 4.79$ mm). The same structure was also used to demonstrate negative refraction in PCs.\[31\] Negative refraction and

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**Figure 4.** Transmission spectra of SRRs, wires, and composite metamaterials (CMMs), i.e., SRRs and wires: a) experiment and b) simulation. Reprinted with permission from [14]. Copyright 2004 Optical Society of America.

**Figure 5.** Measured transmission (solid lines) and reflection (dashed lines) spectra. In each row of this “matrix”, an electron microscopy image of the sample is shown on the right-hand side. The two polarization configurations are shown on top of the two columns. (A,B) ($a = 450$ nm), (C,D) ($a = 600$ nm), and (E,F) ($a = 900$ nm) correspond to nominally identical SRRs, while (G,H) ($a = 600$ nm) correspond to closed rings. The combination of these spectra unambiguously shows that the resonance at about 3 $\mu$m wavelength (highlighted by the gray areas) is the LC (LC: inductor–capacitor) resonance of the individual split-ring resonators. Only the left polarization excites the magnetic resonance. Reprinted with permission from [20]. Copyright 2004 American Association for the Advancement of Science.
focusing in PCs in the microwave regime has also been demonstrated by Parimi and co-workers\[33\] using metallic PCs. Parimi and co-workers\[33\] as well as Parazzoli et al.\[34\] used planoconcave lenses fabricated from a PC and an LHM, respectively, to demonstrate focusing of a plane wave. An inverse experiment\[33,34\] in which a plane wave is produced from a point source placed at the focal point of the lens was also performed. While most of the experiments in PCs\[31–33\] were performed at microwave frequencies, the same structures scaled at optical frequencies must have much smaller losses than the LHMs, which are based on metallic elements. Nevertheless, only two experimental demonstrations of negative refraction in the near-IR frequency region have been made so far in PCs (in PCs made of GaAs\[35\] and Si-polyimide\[36\]).

1.3. Parallel Metallic Slabs and Negative Index of Refraction

Recent theoretical work\[37–39\] concerning attempts to achieve a magnetic response from metallic elements has shown that pairs of finite-length (short) slabs would not only be able to replace the SRRs, but could possibly also lead to negative $n$ directly without the need for additional metallic wires. The condition for obtaining simultaneously negative $\varepsilon$ and $\mu$ by pairs of finite metallic slabs is very restrictive. Recent experiments\[40–42\] have shown evidence of negative $n$ at terahertz frequencies employing pairs of finite slabs. The observed negative $n$, though, was most probably due to the significant imaginary parts of $\varepsilon$ and $\mu$\[43\]. These also lead to a dominant imaginary part of $n$ and thus to a rapid attenuation of EM waves, which makes such metamaterials inapplicable.

1.4. Polaritonic Photonic Crystals

An alternative approach for fabricating metamaterials that would have both negative $\varepsilon$ and $\mu$, and therefore negative $n$, is to use PCs composed of polaritonic materials\[44–47\]. O’Brien and Pendry\[45\] have shown that a 2D square PC of circular ferroelectric rods has a resonance in $\mu$ in the millimeter wavelength range. Huang et al.\[46\] found that a 2D PC composed of polaritonic materials behaves as an effective medium with negative permeability in the micrometer wavelength range. The resonance in $\mu$ in such a medium is due to the large values of $\varepsilon(\omega)$ attained near the transverse phonon\[46\] frequency, $\omega_T$. Shvets\[47\] has suggested that the metallic behavior ($\varepsilon < 0$) of polaritonic materials above $\omega_T$, combined with a PC, which gives $\mu < 0$, can be used to construct LHMs with $\varepsilon$ and $\mu$ simultaneously negative. There are no experiments demonstrating these very interesting ideas yet.

1.5. Nonlinear Left-Handed Materials

While most of the work on LHMs has been done in the linear regime, where both the magnetic permeability and the dielectric permittivity are assumed to be independent of the intensity of the EM field, Kivshar and colleagues\[48,49\] were the first to study the nonlinear properties of LHMs, with some very interesting results. The combination of NIMs with other active or nonlinear materials is a very interesting area of research and needs to be pursued. It might lead to new NIMs with reduced losses at optical wavelengths, broader bandwidths, and other features not possible with passive NIMs. Likewise, the combination of NIMs with various types of nonlinear materials will result in nonlinear LHMs, which have already been predicted to offer new and unusual properties such as soliton formation, second harmonic generation, bistability, phase conjugation, and phase matching. Finally, methods to switch and modulate LHMs can possibly be implemented by combining LHMs with other materials whose EM parameters can be dynamically tuned by the application of external electric or magnetic fields.

2. Electric and Magnetic Response of Metamaterials

As discussed in Section 1.1, the first experimental materialization of Pendry’s ideas was made by Smith et al. in 2000\[2\] since then, various new samples have been prepared (composed of SRRs and wires), all of which have been shown to exhibit a pass band in which it was assumed that $\varepsilon$ and $\mu$ are both negative. This assumption was based on independently measuring the transmission, $T$, of the wires alone, and then the $T$ of the SRRs alone. If the peak in the combined metamaterial composed of SRRs + wires were in the stop bands for the SRRs alone (which corresponds to negative $\mu$) and for the wires alone (which was thought to correspond to negative $\varepsilon$), the peak was considered to be left-handed (LH). Further support for this interpretation was provided by the demonstration that some of these materials exhibited negative refraction of EM waves.\[3\]
Subsequent experiments\textsuperscript{[50]} have reaffirmed the property of negative refraction, giving strong support to the interpretation that these metamaterials can be correctly described by negative permeability, because of the SRRs, and negative permittivity, because of the wires. However, as was shown by Koschny et al.,\textsuperscript{[51]} this is not always the case, as the SRRs also exhibit a resonant electric response in addition to their magnetic response, which was first described by Pendry et al.\textsuperscript{[4]} and will be analyzed in the last part of this section. The electric response of the SRRs, which is demonstrated by closing their air gaps (thereby destroying their magnetic response), is identical to that of cut wires, and it is added to the electric response ($\varepsilon$) of the wires. The result is that the effective plasma frequency, $\omega_p$, of the combined system of wires and SRRs (or closed SRRs) is always lower than the plasma frequency of the wires only, $\omega_p$, as shown in Figure 7. In Figure 8, one can see that, by closing the gaps of the SRRs, the transmission dip at around 4 GHz disappears, while the rest of the spectrum remains almost unchanged. This shows that the 4 GHz dip is magnetic in origin and it is due to the inductor–capacitor (LC) character of the SRR (see below), and also that the closing of the SRR gaps does not affect all the other aspects of its response (the last is valid only if the SRR has mirror symmetry with respect to the incident electric field). With this consideration and the analytical expressions for $\varepsilon$ and $\mu$\textsuperscript{[51]} that stem from it, one is able to reproduce all the low-frequency $T$ and reflection ($R$) characteristics of LHMs. Even the minor details in $T$ and $R$ observed in the simulations can be analytically explained. Moreover, an easy criterion for deciding if an experimental transmission peak is LH or right-handed (RH) can be achieved: if closing the gaps of the SRRs in a given LH structure removes the peak close to the position of the SRR dip from the $T$ spectrum, this is strong evidence that the $T$ peak is indeed LH. If the gap above the peak is removed, the peak is most likely RH. This criterion is very valuable in experimental studies\textsuperscript{[14]} where one cannot easily obtain the effective $\varepsilon$ and $\mu$. The criterion has been used experimentally, and it was found that some $T$ peaks that were thought to be LH turned out to be RH\textsuperscript{[13]}.

It is well known from elementary electromagnetism that a magnetic dipole can be realized by the circulating current of a closed metallic loop, such as a closed SRR (CSRR), which leads to a magnetic moment, $m$, with magnitude given by the product of the current and the area of the CSRR and direction perpendicular to the plane of the SRR. Therefore, CSRR behaves as an inductor, storing magnetic energy $U = mB = LI^2/2$, where $L$ is the self-inductance of the loop, $B$ is the magnetic field ($B = \mu_0 H$), $I$ is the current in the loop, and $\mu_0$ is the free-space magnetic permeability. If a CSRR is combined with a capacitor with capacitance $C$ then one obtains an LC circuit with a resonance frequency $\omega_{LC} = 1/\sqrt{LC}$. Such a capacitor can be realized by making a cut in the ring, leading to a normal SRR. Thus, the SRR acts like an EM resonator, producing at $\omega_{LC}$ resonant circular currents leading to resonant magnetization, i.e., resonant effective permeability.

Figure 9 shows the analogy of a conventional LC circuit and a metallic single-ring SRR on a dielectric substrate. The order of magnitude of the resonance frequency for such an SRR can be easily estimated by considering the SRR capacitance concentrated in the area of its gap, treating the gap as a parallel plate capacitor with $C = \varepsilon_0 /d$ (where $w$ is the width of the metal, $d$ is the width of the gap of the capacitor, and $t$ is the metal thickness)—see Fig. 9B for these definitions—including the free-space permittivity and the relative permittivity of the material in the gap of the capacitor, and using for the inductance the formula for the inductance of a solenoid, i.e., $L = \mu_0 LI/t$, where $l$ is the size of the SRR. Thus, $\omega_{LC} = 1/\sqrt{LC} = (\varepsilon_0 /\varepsilon_C) \sqrt{d/w}$, where $c_0$ is the velocity of light in vacuum. The corresponding free-space wavelength, $\lambda_{LC} = (2\pi) /\sqrt{LC} \sqrt{d/w}$, is proportional to the size of the SRR. In principle, $\lambda_{LC}$ can be much larger than the size of the SRR. This is a fundamental difference between LHMs and the other important class of EM metamaterials, PCs, where, as was mentioned in Section 1.2, the frequencies of operation correspond to wavelengths of the same order of magnitude as the size of the unit cell. Since in LHMs the operation wavelength, $\lambda_{LC}$,
can be much larger than the size, $a$, of the unit cell, an LHM can very accurately be considered a homogeneous effective medium and described using effective medium theories, which greatly simplify its description and facilitate the physical understanding of its main features.

Indeed, there is a significant amount of theoretical and numerical work where the homogeneous effective medium assumption was used for the development of a retrieval procedure, which was applied to obtain the effective $\varepsilon$ and $\mu$ of a metamaterial from calculated reflection and transmission amplitudes. This procedure confirmed\footnote{52–56} that a medium composed of SRRs and wires could indeed be characterized by effective $\varepsilon$ and $\mu$ with negative real parts over a finite frequency band, and by an $n$ also with a negative real part.

One can easily obtain a simple expression for the frequency dependence of the effective magnetic permeability $\mu(\omega)$ for a lattice of SRRs, assuming an incident EM field propagating parallel to the SRRs plane with magnetic field perpendicular to the SRR plane and an electric field parallel to the sides of the SRRs that do not have cuts. Under these conditions, and according to the Kirchhoff loop rule, the self-induction voltage of the inductance $L$, $U_L$, plus the voltage drop across the capacitance $C$, $U_C$, equals the voltage induced by the external magnetic flux, $U_{\text{ind}}$

$$U_L + U_C = U_{\text{ind}} \quad \text{or} \quad LI + (1/C)\frac{d\phi}{dt} = U_{\text{ind}} = -\dot{\phi} \tag{1}$$

where $\phi$ is the external magnetic flux, $\dot{\phi} = \mu_0 I H$, where $I$ is the current, $H$ is the external magnetic field, which has the harmonic time dependence $H = H_0 e^{-i\omega t}$, and the dot above the symbols denotes the time derivative. Taking the time derivative of Equation 1, we obtain

$$\ddot{I} + \frac{I}{LC} = \frac{U_{\text{ind}}}{L} = +\omega^2 \frac{\mu_0 I^2}{L} H_0 e^{-i\omega t} \tag{2}$$

The obvious solution is $I = I_0 e^{-i\omega t}$, where

$$I_0 = -\frac{\omega^2 (\mu_0 I^2 / L)}{\omega^2 - 1 / LC} H_0 \tag{3}$$

and we can easily obtain the individual SRR magnetic dipole moment, area $\times$ current $= \ell I$, and the magnetization $M = (N_{LC} / V)^2 I$, where $N_{LC}$ is the number of LC circuits and $V$ is their corresponding volume, i.e., $N_{LC} / V = 1 / (a_\perp a \perp)$, where $a_\perp > \ell$ is the lattice constant in the SRR plane and $a \perp > \ell$ is the lattice constant in the direction normal to the SRRs. Finally, using $M = \chi_m(\omega) H$, $\mu(\omega) = 1 + \chi_m(\omega)$, and $L = \mu_0 \ell / t$, with $\chi_m$ the magnetic susceptibility, Equation 4 is obtained as

$$\mu(\omega) = 1 + \frac{F \omega^2}{\omega_{LC}^2 - \omega^2} = 1 - \frac{F \omega^2}{\omega^2 - \omega_{LC}^2} \tag{4}$$

Apart from the $\omega^2$ in the numerator, this represents a Lorentz oscillator resonance for a magnetic atom. Here, we have lumped the various parameters into the dimensionless quantity $F = F_{\text{dip}} / (a_\perp a \perp)$, which is less than one. As is done in most cases, all the losses and the scattering mechanisms can be lumped into a damping factor $\Gamma_m$ added in the denominator of Equation 4. Note that as $\omega_{LC} < \omega < \omega_{LC} \sqrt{1 - F}$, $\mu(\omega)$ is always negative.

At the heart of the metamaterials concept is a fundamental physics approach in which a continuous material, described by the relatively simple EM parameters $\varepsilon$ and $\mu$, conceptually replaces an inhomogeneous collection of scattering objects. In general, the continuous material parameters are tensors and are frequency dependent, but nevertheless represent a considerable reduction in complexity for describing wave-propagation behavior.

It has been found that the averaged response of artificially structured metamaterials follows well-known forms of response that occur in conventional materials. A common means of describing material properties, for example, is in terms of the Drude–Lorentz model, in which the details of a material are replaced conceptually by a collection of harmonically bound charges—either electric or fictitious magnetic. The bound charges are displaced by incident (electric or magnetic) fields, giving rise to a polarized medium. If the polarization is linearly related to the applied fields, then Max-

Figure 9. Illustration of the analogy between a usual LC circuit (A) and a SRR (B). C) Electron microscopy image of a fabricated structure, a gold SRR (metal thickness $t=20$ nm) on a glass substrate. Reprinted with permission from [20]. Copyright 2004 American Association for the Advancement of Science.
well’s equation combined with the oscillator model yields the well-known effective material parameters,

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_d^2 + i\Gamma_\varepsilon \omega} \\
\mu(\omega) = 1 - \frac{\omega_m^2}{\omega^2 - \omega_m^2 + i\Gamma_m \omega}
\]

These are the standard Drude–Lorentz forms for the permittivity and permeability. Their form stems from the universal resonant response of a harmonic oscillator to an external frequency-dependent perturbation. Note that for each resonance three parameters enter: the plasma frequency, \(\omega_p\), the resonant frequency, \(\omega_r\), and a damping factor, \(\Gamma\). These parameters are indexed with an “e” for electric or an “m” for magnetic response. The Drude–Lorentz forms correctly describe the EM response of materials over frequencies that range from microwaves to optical or UV, thus providing a convenient language uniting the description of materials over all frequency ranges. The free-electron contribution in metals corresponds to \(\omega_{m0} = 0\). We point out in passing that the concept of resonance is also crucial for the existence of PCs. The difference from the LHMs is that in PCs the resonance is purely geometrical and occurs when the wavelength inside the material is comparable to the period of the structure. In contrast, LHMs could have characteristic structural lengths much smaller that the wavelength, thus permitting their effective homogeneity description. This property of LHMs could be used in the miniaturization of devices.

Metamaterials based on strong artificial resonant elements can also be described quite efficiently with formulas similar to the Drude–Lorentz forms. In fact, two types of artificial elements have served as the basic building blocks for a wide variety of metamaterials. These structures are the metallic wire structure, which provides a predominantly free-electron response to EM fields, and the SRR structure, which provides a predominantly magnetic response to EM fields.

Materials with negative \(\varepsilon\) are well known, and have been investigated for many years. In naturally occurring materials, the resonances that give rise to the Drude–Lorentz forms occur within generally restricted frequency ranges. Electric resonances, for example, tend to occur in the high terahertz frequencies or much higher and result from phonon modes, plasma-like oscillations of the conduction electrons, or other fundamental processes. Magnetic resonances generally occur in inherently magnetic materials, and are associated with such processes as ferromagnetic or antiferromagnetic resonance. These resonances tend to die out in the higher gigahertz frequencies, and are absent in all but a few specialized systems at terahertz frequencies.

While conventional material responses appear to be restricted, this is not a fundamental limitation, and thus metamaterials can be designed that have either electric or magnetic resonances where there are no equivalent existing materials. Recent work, including our contributions, has demonstrated that electric and magnetic resonances can be situated at any frequency, up to terahertz frequencies, in metamaterial structures. In particular, by combining electric and magnetic structures, it is possible to arrive at a material with a frequency band over which both \(\varepsilon\) and \(\mu\) are simultaneously negative. For such a material, \(n\), determined by taking the square root of the product \(\mu\varepsilon\), is real, indicating the material is transparent to radiation. However, it has been shown that the correct choice for the sign of the square root is negative when both \(\varepsilon\) and \(\mu\) are negative. Thus, materials for which \(\varepsilon\) and \(\mu\) are both negative can also be characterized as NIMs.

3. Saturation of the Magnetic Response of SRRs at Optical Frequencies

There is a sustained effort in the community to push the operation frequency of the metamaterials deeper and deeper into the terahertz region, ultimately to reach optical frequencies, since fiber telecommunications and optics operate near and at this frequency range. However, LHMs require both negative \(\varepsilon\) and negative \(\mu\). While it is easy to find negative-permittivity materials (e.g., ordinary metals), the same is not true for negative permeability since natural materials do not exhibit any magnetic response at such high frequencies, i.e., they have \(\mu = 1\). SRRs may offer an answer to the demand for high-frequency negative \(\mu\) since, at least for frequencies up to several terahertz, the magnetic resonance frequency scales reciprocally with the structural size. At higher frequencies, however, this linear scaling breaks down, as was shown by Zhou et al. The reason is the following: there are in general two velocity-dependent contributions to the energy of a metallic wire in which an electric current, \(I = \omega e m \nu_c\), is flowing (\(\nu_c\) is the cross section of the wire—see Figure 9B, \(n_e\) is the concentration of free electrons, and \(\nu_c\) is their mean velocity). One is the magnetic energy, \(L_m F^2/2\), and the other is the kinetic energy of the free electrons, \(E_k = N \nu_m \nu_c^2 / 2\), where the volume, \(V\), of the wire is \(V = \pi l^2\), with \(l = 4(l-w)\) (see Fig. 9B) being the length of the wire. To compare these two contributions, we can re-express the kinetic energy in terms of \(I\) instead of \(\nu_c\): \(E_k = L_m F^2/2\), where \(L_m = l m / \pi \nu_c^2 r_n\) (\(F = \nu_c / (1/\nu_c^2 r_n)\)). Thus, for a wire making a ring like the one in Figure 9B, when all the sizes (\(w, d, L\)) scale in proportion to the unit cell size, \(a\), the effective inductance \(L_e\), corresponding to \(E_k\), scales inversely proportional to \(a\), in contrast to the magnetic inductance, \(L_m\), which is proportional to \(a\). The ratio \(L_m / L_e\) is of the order of \(10 \omega l / \lambda_m^2\), with a typical value of \(\lambda_m\) being around 100 nm. Thus, for \(\sqrt{\omega l}\) considerably larger than 100 nm, the kinetic energy, \(E_k\), of the electrons is negligible in comparison with the magnetic energy; but for \(\sqrt{\omega l}\) smaller than 100 nm, \(E_k\) becomes appreciable and may dominate as \(\sqrt{\omega l}\) becomes smaller and smaller.
The capacitance $C$ of the SRR also scales in proportion to the size of the SRR, so the magnetic resonance frequency has a size dependence given by

$$\omega_{LC} = \frac{1}{\sqrt{(L_m + L_e)C}} \propto \frac{1}{\sqrt{\text{size}^2 + \text{const.}}} \tag{6}$$

For very large structures, the resonance frequency is inversely proportional to the size of the SRR, while for small structures $\omega_{LC}$ approaches a constant. To get a rough estimate of the saturation value of $\omega_{LC}$, we take into account the fact that the capacitance $C$ is given by $C = \varepsilon_0 w d$. By using the expression of the metal plasma frequency, $\omega_p^2 = n_e e^2 / \varepsilon_0 m_e$, we find that the saturation magnetic resonance frequency is approximately given by

$$\omega_{LC}^{\text{max}} = \frac{1}{\sqrt{\varepsilon_0 C}} = \omega_0 \sqrt{\frac{d}{4l}} \tag{7}$$

This saturation frequency is further reduced by the dielectric environment and by the skin effect,[57] which were neglected in our rough estimate. Zhou et al. investigated the limits of the resonant magnetic response for single-ring multicut SRR designs, shown in Figure 10, up to optical frequencies.[57] It was shown (see also Fig. 11) that the breakdown of linear scaling due to the free-electron kinetic energy occurs for frequencies above 100 THz. Well above the linear scaling regime, the resonance frequency saturates, while the amplitude will lead to the undesirable excitation of the magnetic resonance by the electric field. To avoid this coupling, more symmetric SRRs should be used. Koschny et al.[26] proposed a 3D isotropic LHM design, shown in Figure 12, based on single-ring four-gap SRRs that allows LH behavior for any direction of propagation and any polarization of the EM wave. Using numerical transfer matrix simulations, they verified the isotropic transmission properties of the proposed structure. Their data show excellent agreement with results expected for a homogeneous slab with the corresponding negative $\varepsilon$ and $\mu$. No 3D isotropic LHM has been fabricated to date, and it is a challenge to be built, even at microwave frequencies.

4. Negative-Index Materials using Simple, Short-Slab Pairs

All NIM implementations to date have utilized the topology proposed by Pendry consisting of SRRs and continuous wires. Many groups have been able to fabricate NIMs with $n = -1$ with losses of less than 1 dB cm$^{-1}$ in the gigahertz regime.[2,3,9–18,31–33,50,58,59] Recently, different groups have indirectly observed negative $\varepsilon$ in the terahertz region.[19–23,40] In most of the terahertz experiments,[19,20,40] only one layer of SRRs was fabricated on a substrate, and the transmission was measured only for propagation perpendicular to the plane of the SRRs, exploiting the coupling of the electric field to the magnetic resonance of the SRR via asymmetry.[111] This way it is not possible to drive the magnetic permeability negative. Also, a negative $n$ with a small imaginary part has not yet been observed in the terahertz region. One reason is that it is very difficult to measure—with the existing topology of SRRs and continuous wires—the $T$ and $R$ along the direction parallel to the plane of the SRRs. Therefore, there is a need for alternative, improved, and simplified designs that can be easily
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A short-slab pair can behave like an SRR, exhibiting a magnetic resonance resulting in a negative-permeability regime. Moreover, short-slab pairs can in principle give a negative \( \varepsilon \) and a negative \( \mu \) in the same frequency range, and therefore a negative \( n \), without the need for additional continuous wires.\(^{35-39}\) However, recent experiments\(^{23}\) have not shown evidence of negative \( n \) at terahertz frequencies in the short-wire pair cases that were studied. This is in contrast with the claims\(^{[41]}\) that one can achieve negative \( n \) at terahertz frequencies. The negative \( n \) obtained\(^{[40-42]}\) at terahertz frequencies is most probably due to the large imaginary parts of \( \varepsilon \) and \( \mu \).\(^{[43]}\)

Very recent work\(^{[60,61]}\) introduced new designs of short-slab-pair-based metallic structures in order to obtain a negative \( n \) in the microwave regime. The basic structure of a single unit cell of this NIM was built from H-shaped slabs, and is as shown in Figure 13: the short-slab pair consists of a pair of metal patches separated by a dielectric spacer of thickness \( \varepsilon_{\text{r}} \). For an EM wave incident with a wave vector and field polarization as shown in Figure 13, the short-slab pair acquires not only a magnetic resonance resulting in a negative \( \mu \)\(^{[34,41,61]}\) but also, simultaneously, an electric resonance with a negative \( \varepsilon \).

The magnetic resonance originates from antiparallel currents in the slabs of the pair, resulting in opposite charge accumulating at the corresponding ends; the electric resonance is due to the excitation of parallel currents in the slabs of the pair, with same-sign charge accumulating at the corresponding ends of both wires. Repeating this basic structure periodically in the \( x \)-, \( y \)-, and \( z \)-directions would result in a NIM structure.

Transmission and reflection properties of a single-layer structure were measured over the frequency range 13–18 GHz using a network analyzer (HP8510) and a pair of standard gain horn antennas serving as source and receiver. In the transmission measurements, the microwaves were incident normal to the sample surface. This is a tremendous simplification relative to conventional SRRs and wires, where the incident EM waves have to propagate parallel to the sample surface. With the conventional orientation of the SRRs, it is almost impossible to do this type of measurement at the terahertz region, since only single-layer samples are usually fabricated.\(^{[19,20]}\) In both measurements, the electric field of the incident wave was polarized parallel to the long dimension of the slabs. (For perpendicular polarization the transmission at the resonance regime was nearly 100 %, independent of the frequency, and the reflection was essentially zero.)

Using the transmission and reflection results from a single layer, we can extract the effective \( n \) that would result if a periodic multilayer sample were built using the single-layer structure as a building block. The details of the numerical retrieval procedure have been described in detail elsewhere.\(^{[44-49]}\) The extracted \( n \) is shown in Figure 14 and the extracted permittivity and permeability are shown in Figure 15. The plots show that the real part of the permittivity (see Fig. 15A) is negative over most of the measured range. The real part of the permeability is negative over a band near 16 GHz for the simulation and the experiment. The extracted real part of \( n \) is negative\(^{[41]}\) over a narrow band at 16 GHz. The ratio of the imaginary part of \( n \) to the real part of \( n \) is 1/3, which means that we have LH propagation with \( \varepsilon \), \( \mu \), and \( n \) negative.

![Figure 12](image1)

**Figure 12.** The design of a fully symmetric unit cell of an isotropic SRR (a), and an LHM based on this symmetric SRR design (b). The interfaces are parallel to the left and right SRR. The metal of the four-gap SRR (dark gray) and the continuous wires (medium gray) is silver using a Drude-model permittivity around 1 THz. The SRR gaps are filled with a high dielectric constant material (light gray) with a relative permittivity \( \varepsilon_{\text{gap}} = 300 \) to lower the magnetic-resonance frequency. Reproduced with permission from \(^{[26]}\). Copyright 2005 American Physical Society.

![Figure 13](image2)

**Figure 13.** a) Schematic representation of one unit cell of the slab-pair structure. b) Photograph of a fabricated microwave-scale slab-pair sample.
These results clearly show the viability of using short-slab pairs to build NIMs, either combined with additional continuous wires or not. It is likely that modifications of the basic structure presented in Figure 13 may improve or alter the NIM properties. Also, slab-pair arrangements with significantly different geometries may lead to NIMs. The relative ease of fabricating slab-pair structures may hasten the development of NIMs working at optical wavelengths.

5. Concluding Remarks

NIMs have rapidly achieved widespread recognition as they allow previously unavailable solutions of Maxwell’s equations. As such, NIMs represent a striking example of the utility of metamaterials. Yet, although remarkable physical phenomena have been predicted for NIMs, including evanescent-wave refocusing (leading to “perfect lensing”), nearly aberration-free lenses, reversed Doppler shifts, and reversed Cerenkov radiation, the limitations of negative materials must be kept in mind. For example, it has been suggested that a surface with $\varepsilon = \mu = -1$ can be reflectionless. This statement, however, is only true in a steady-state sense; if a wavefront from free space impinges on such a surface, reflections associated with transients will, in fact, occur until the steady-state solution is reached.

Efforts over the past several years, including ours, have been instrumental in proving that negative-index metamaterials can be designed, fabricated, and characterized. Negative refraction, in steady-state experiments, has now been demonstrated many times. Experiments showing image resolution beyond the diffraction limit via a negative-index slab have been published. Thus, the work of several groups in the last four years has placed negative $n$ on solid ground: We are now in a position to move forward and further develop the materials and methods that will make these novel materials useful.

In particular, we believe that the uniqueness and novelty of LHMs or NIMs are the following:

i) The ability to match the vacuum impedance; this is a unique property of NIMs with many applications (e.g., stealth technology), stemming from the fact that no reflection is created at an interface separating a medium with $\varepsilon = \mu = 1$ (e.g., air) and a LHM with $\varepsilon = \mu = -1$.

ii) The possibility of creating patterns that allow for coupling with the magnetic component of an EM field without the presence of any magnetic material; this is a new capability of fundamental importance, especially in the terahertz region where no natural magnetic resonance exists.

iii) The possibility to miniaturize devices and components such as antennas and waveguide structures, especially at long $\lambda$; this is very important because of potential system weight and size savings. NIMs provide a platform for a revolutionary change in the design of subwavelength devices.

iv) Finally, the negative index of refraction and the subwavelength resolution capability; this opens up the possibility of new applications in optics and communications.

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In this scenario however, the imaginary part of the frequency becomes large, because in a complex material, it is possible to have simultaneously being negative. This is the case if the imaginary part of the frequency is sufficiently large. In this case, the imaginary part of the frequency is dominated by the negative imaginary part of the frequency at the high-frequency side, where we can observe the LH behavior. In our experiments [15,16], although we have considered LH behavior in our experiments, we have not been able to simultaneously being negative. In this case, the imaginary part of the frequency is sufficiently large. In this case, the imaginary part of the frequency is simultaneously being negative.