# Effect of disorder on magnetic resonance band gap of split-ring resonator structures

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**Abstract:** We investigated the influence of periodicity, misalignment, and disorder on the magnetic resonance gap of split-ring resonators (SRRs) which are essential components of left handed-metamaterials (LHMs). The resonance of a single SRR which is induced by the split is experimentally demonstrated by comparing transmission spectra of SRR and closed ring resonator. Misaligning the SRR boards do not affect the magnetic resonance gap, while destroying the periodicity results in a narrower band gap. The disorder in SRR layers cause narrower left-handed pass band and decrease the transmission level of composite metamaterials (CMMs), which may significantly affect the performance of these LHMs.

**OCIS codes:** (160.4670) Optical materials; (260.5740) Physical Optics, resonance.

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## References and links

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## 1. Introduction

The realization of media with negative effective index of refraction ($n_{\text{eff}}$) resulted in discovery of novel aspects of electromagnetism and made a great impact on physics community. Inspired by the proposal of Veselago in 1968 [1], an intuitive approach was taken to construct a composite medium, where the components have negative permittivity ($\varepsilon(\omega) < 0$) and negative permeability ($\mu(\omega) < 0$), simultaneously over a frequency range. In this case, the wavevector of the propagating electromagnetic (EM) wave is real and $n_{\text{eff}}$ is negative [2,3]. By the pioneering works of Pendry *et al.*, [4] a structure consisting of periodically arranged metallic wires is shown to exhibit plasma cut-off frequency, $\omega_p$, in the microwave regime, below which the material is opaque. Following this, metallic split-ring resonator (SRR) structures (Fig.1) are proposed and demonstrated to have $\mu(\omega) < 0$ near the magnetic resonance $\omega_{mp}$ [5]. The experimental verification of negative refraction is reported shortly after [6-8], supporting the existence of $n_{\text{eff}} < 0$ medium. These structures are usually called composite metamaterials (CMMs) [2] or left-handed metamaterials (LHMs), attributing to the left-handed coordinate system formed by the EM wave components in the medium. As the studies matured, a solid understanding of the conditions for left-handed behavior has started to build up [9-12]. Particular emphasis is given to the fact that SRRs have not only magnetic response but also dielectric response, which causes $\omega_p$ of the composite medium shift to lower frequencies and differ substantially from that of the wire-only medium [10-12].

As the CMM structures are evolving through different designs and dimensions, their composite nature consisting of SRR and wire elements remained essentially the same. In one dimension (1D), a CMM is constructed simply by stacking the wire and SRR layers alternatingly. While the studies concerning CMMs are gearing towards device applications, we found that the effect of disorder, that can occur during fabrication or stacking of these structures, on the transmission properties of left-handed CMMs is rather unexplored. In a previous work, [3] the problem of misaligment at microwave frequencies is adressed in explaining the left-handed transmission band shift. Evidently, disorder will be a more significant problem for submicron scale structures operating in the near and far infrared. Therefore, the effect of misalignment and disorder needs to be investigated for determining the restrictions imposed on the LH operation of CMMs.

In this article, we aim to close this gap by presenting a systematic study of disorder effects on the transmission properties of SRRs and CMMs operating in the microwave regime. Even though scaling is not applicable in a strict sense, small scale structures may benefit from the conclusions drawn here as the physics remain essentially the same. We focus on the disorder of SRR layers, as the $\mu(\omega) < 0$ gap defines where the LH ($n_{\text{eff}} < 0$) transmission occurs.

The paper is organized as follows. We first describe what type of disorder can occur in SRR structures. The experimental measurements of the SRR transmission spectra are presented where disorder is introduced systematically to the SRR layers. We then discuss the effect of disorder in the SRR layers of CMM. We conclude the paper with brief remarks.

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## 2. Disorder in split-ring resonator medium

It is well-known that a SRR structure (Fig. 1(a)) is resonant around magnetic plasma frequency ($\omega_{mp}$) [2,5,13], induced by the currents and the split which imitates magnetic poles. Closing the splits as in Fig. 1(b) will destroy the magnetic resonance of SRRs and therefore $\mu < 0$ gap will disappear. Similarly, a periodic medium of SRRs and closed-SRRs (CSRRs) can be used respectively to distinguish the magnetic resonance gap of SRR structures [9].

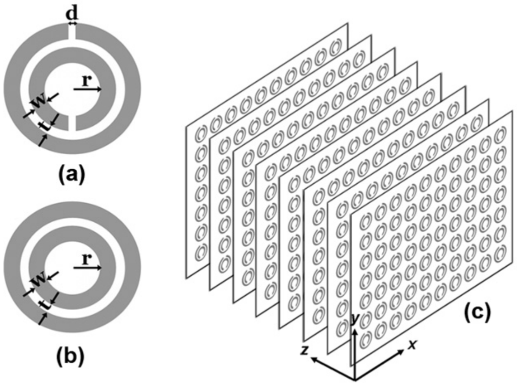

Fig. 1 Schematic drawings of (a) a single split-ring resonator, (b) a single closed ring resonator, (c) periodic arrangement of split ring-resonators on dielectric boards.

We measured the transmission through a single unit cell of SRRs and CSRRs to determine the $\omega_{mp}$ using monopole antennas (inset of Fig. 2). The dimensions of the SRR are $d = t = 0.2$ mm, $w = 0.9$ mm and $r = 1.6$ mm as shown in Fig. 1(a). The circuit board has a thickness 1.6 mm and dielectric constant of $\varepsilon = 4.4$. Figure 2 displays the measured transmission spectra for single SRR and single CSRR structures. Evidently, SRR structure is resonant around 3.83 GHz.

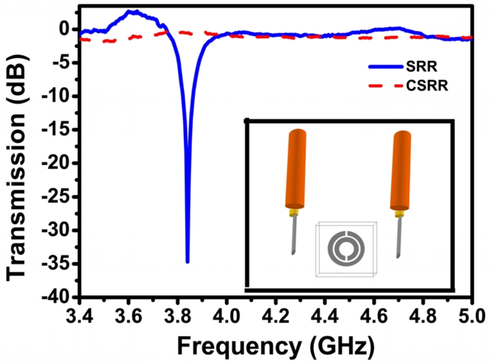

Fig. 2. Measured transmission spectra of a single split-ring resonator (blue) and closed ring resonator (red). [Inset] Schematics of experimental setup where two monopole antennas are used to measure a single SRR cell.

SRR units are then arranged periodically to obtain a negative permeability medium. The periodic SRR structure (Fig. 1(c)) is made of $N_x = 10$, $N_y = 15$, and $N_z = 24$ unit cells, with lattice spacings $a_x = a_y = 8.8$, and $a_z = 6.5$ mm. Free space transmission measurements are performed by using a HP 8510C network analyzer and microwave horn antennas. The incident electromagnetic (EM) wave propagates along the $x$ direction, while **E** is along $y$ direction, and **H** is along $z$ direction for all measurements (Fig. 1(c)).

The disorder can be introduced into the SRR structures in several ways. One is the inter-plane disorders which can occur during stacking: In this case, the SRRs within each plane (board) are periodic in the *x* and y directions, but the planes themselves are shifted arbitrarily. Inter-plane disorders can also be named as misalignments. Retaining the periodicity within a board is easier during the fabrication phase, but arranging all boards periodically could be a problem in smaller scales. In view of the reference structure in Fig. 1(c), which is periodic and ordered, the top plane of Fig. 3 displays three different kind of inter-plane disorders: (a) SRR medium non-periodic along the *z* direction by randomizing the interplane distance $a_z \pm \delta_z$ where $\delta_z \leq \lambda/4$.; (b) Misalignment along the propagation direction, *x*; (c) Increased misalignment along *x*. Misalignment along the *x* direction is achieved by shifting the SRR boards by $|\delta_x| \leq \lambda/8$, where for increased misalignment the parameter is taken as $|\delta_x| \leq \lambda/2$.

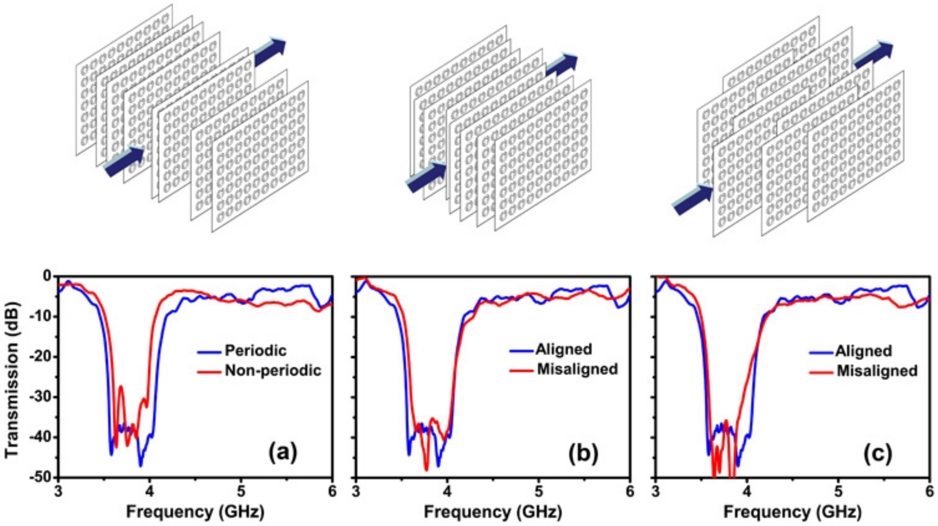

Fig. 3. [Top panel] Schematic drawings of disordered split-ring resonator media with (a) disorder in *z* direction with randomizing the inter-plane distance $a_z \pm \delta_z$ where $\delta_z \leq \lambda/4$, (b) disorder in *x* direction with misalignment parameter $|\delta_x| \leq \lambda/8$, (c) increased disorder in *x* direction with parameter $|\delta_x| \leq \lambda/2$. [Bottom panel] Measured transmission spectra of periodic and ordered SRR medium (blue) and corresponding disordered split-ring resonator media (red).

Arranging SRR structures periodically in all directions increases the coupling between the SRRs, therefore, a wider band gap in the transmission spectrum is observed compared to that of single SRR. Periodic and ordered structure has a magnetic resonance band gap between 3.55 - 4.10 GHz (blue line in Fig. 3). A similar band gap (3.6 - 4.0 GHz) is observed for the non-periodic SRR medium (Fig. 3(a)). Stacking SRR boards non-periodically still keeps the magnetic resonance gap, but the band width gets narrower. This effect can be explained by the reduced coupling between SRR boards. Measured transmission spectra for misalignment (Fig. 3(b)) and increased misalignment (Fig. 3(c)) of SRR boards show that band gap is still present in both cases. Hence, magnetic resonance frequency of SRRs is not influenced by misalignment in *x* and *z* directions to the extent of disorder we have introduced.

The other type of disorder is intra-plane disorder, where we introduce randomness to the positions of SRRs along the *x* and *y* directions within a board. The disorder is introduced as follows. Each SRR on the board with lattice point, $\vec{\mathbf{r}}_n$, where $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ is displaced with $\vec{\mathbf{r}}_n \pm \vec{\boldsymbol{\delta}}_r$. Therefore the degree of disorder can be changed by varying the randomness parameter, $\vec{\boldsymbol{\delta}}_r$. We consider two cases: (a) $|\vec{\boldsymbol{\delta}}_r| \leq a/9$, and (b) $|\vec{\boldsymbol{\delta}}_r| \leq a/5$, which are displayed schematically in top panel of Fig. 4(a) and (b), respectively. Here, $a = a_x = a_y = 8.8$ mm is the

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lattice constant on periodic SRR board. The measured transmission spectra are given in bottom panels.

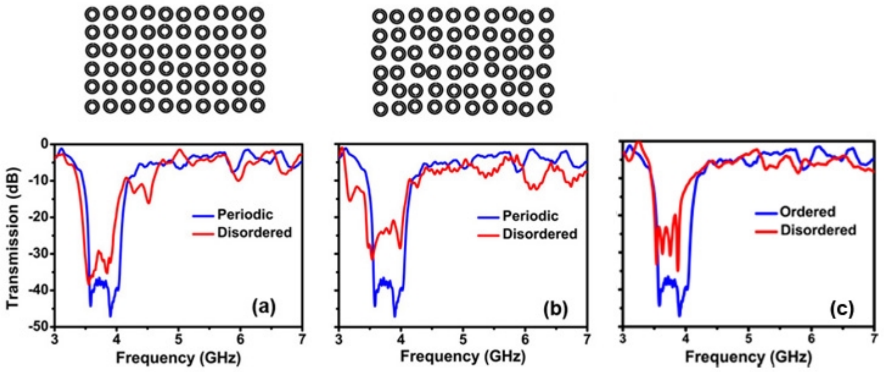

Fig. 4. [Top panel] Schematic drawings of intra-plane disordered SRRs with (a) $|\vec{\boldsymbol{\delta}}_r| \le a/9$, (b) $|\vec{\boldsymbol{\delta}}_r| \le a/5$ where $\vec{\boldsymbol{\delta}}_r$ is the randomness parameter. [Bottom panel] Measured transmission spectra of ordered SRR (blue) and disordered SRR media (red), (c) Comparison of the transmission spectra for ordered and disordered SRRs where the disorder is in all 3 spatial directions.

By introducing disorder, we observe that magnetic resonance band gap still remains, but the transmission inside the band gap is increased with increasing disorder. The average transmission inside the band gap (3.55 - 4.10 GHz) is -40 dB for ordered structure, -32 dB for disordered ($|\vec{\boldsymbol{\delta}}_r| \le a/9$) structure (Fig. 4(a)) and -25 dB for structure with increased ($|\vec{\boldsymbol{\delta}}_r| \le a/5$) disorder (Fig. 4(b)). Each SRR unit has an effect on lowering the transmission and therefore causing a band gap due to magnetic resonance. Adding disorder into SRR structures changes the interaction between the SRRs and this in turn causes a higher transmission inside the band gap.

By combining the inter-plane and intra-plane disorder, one can obtain a structure, disordered in all three spatial directions. Figure 4(c) gives the measured transmission spectra for fully disordered SRR structure obtained by arranging intra-plane disordered SRR boards (Fig. 4(a)) as non-periodic (Fig. 3(a)) and misaligned (Fig. 3(b)) arrays. Magnetic resonance is still present with a relatively higher transmission. A narrower magnetic resonance band gap (3.55 - 3.90 GHz) is observed when compared to ordered SRR structure, and this behavior can be explained by the reduced coupling between SRRs.

The coupling between two or more SRRs is reported to be a complex mechanism and dependent on their particular geometrical arrangement [13]. The dominant coupling occurs between consecutive SRRs along the propagation direction of the EM field which depends strongly on the distance between SRRs along this direction. Thus, deviations from the periodic arrangement along the propagation direction (in our case the $x$ direction) have more impact on the properties of the negative permeability region. On the other hand, the coupling of SRRs in adjacent planes (boards) is not strong, hence the non-periodicity introduced in lateral direction (in our case the $z$ direction) have subtle effects on the magnetic resonance. Our experimental results summarized in Figs 3 and 4 agree with these conclusions. To this end, we add that a band gap can also be formed due to the electric resonance of SRRs and Bragg-like multiple scatterings arising from the periodicity [10]. It is shown that for metallic crystals, Bragg-like scattering is the dominant factor and disorder dramatically changes the transmission characteristics [14]. Our measurements confirm that the magnetic resonance band gap of SRRs is still present even for large amount of disorders. A similar persistent behaviour is reported for Mie resonances in dielectric photonic crystals (PCs) in the presence of disorder [14].

## 3. Disorder in composite metamaterials

We now discuss the effect of incorporating intra-plane disordered SRRs in CMM structures. LH transmission band occurs at frequencies where both $\varepsilon_{\text{eff}}(\omega) < 0$ and $\mu_{\text{eff}}(\omega) < 0$. Periodic CMM is obtained by arranging periodic SRR medium and periodic thin wire medium. The thickness, width, length and periodicity of the wires are 30 μm, 0.9 mm, 13 cm and $a_x = 8.8$ mm, respectively. Figure 5(a) depicts the measured transmission spectra of periodic SRR (blue) and periodic CMM (red) structures. Plasma frequency of the periodic wire medium is around 8 GHz [10]. The combined wire and SRR structure has an effective plasma frequency reduced down to 5.3 GHz due to interaction between SRRs and wires [10]. The left-handed transmission band occurs in the frequency range (3.6–4.1 GHz).

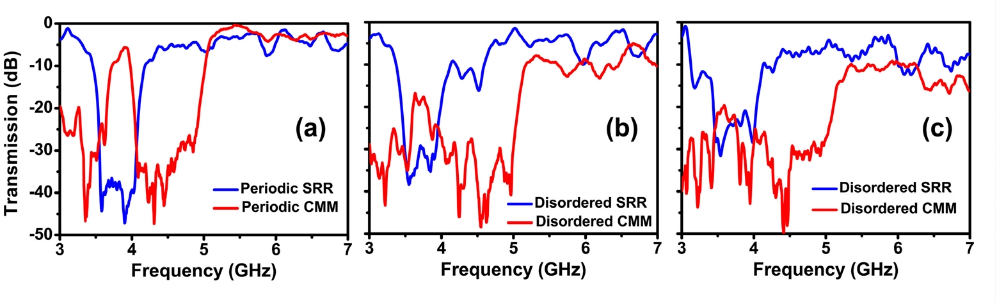

Fig. 5. Measured transmission spectra of periodic and ordered SRR medium (blue) and corresponding disordered split-ring resonator mediums (red).

By combining the intra-plane disordered SRR boards (top plane of Fig. 4) with periodic wire mediums, we obtain a disordered CMM medium. The reason why we did not introduce disorder to the wire structures is the sensitivity of band gap of metallic wires to the disorder. Disorder causes a downward shift of the plasma frequency of wire medium [14], which will obscure the negative permittivity region. Disordered CMM structure is obtained by arranging intra-plane disordered SRR structure (Fig. 4(a)) with the thin wire medium periodically. Figure 5(b) depicts the measured transmission spectrum for disordered CMM. The transmission peak is reduced and the transmission band became narrower. Employing increasingly disordered SRRs (Fig. 4(b)) in CMM narrows the band width and lowers the transmission peak (Fig. 5(c)). Since low transmission and narrow band width for the LH transmission band are major problems of LHMs, disorder in these structures can be a significant factor that affects the performance of LHMs.

## 4. Conclusion

In conclusion, we have demonstrated that the magnetic resonance gap of SRRs persists in the presence of misalignment and disorders. When the inter-plane distance is randomized, the effective medium arising from coupling of SRR boards is degraded that results in a narrower magnetic permeability region. By employing intra-plane disorders, magnetic resonance band gap still exists but with a higher transmission compared to the ordered SRR structure. In contrast to SRR structures, CMM is influenced from the disorders manifested by a lower and narrower left-handed transmission band. As a result, to the extend of disorder investigated here, the low transmission and narrower band width may cause an impaired LH behavior.

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