Delay-time investigation of electromagnetic waves through homogeneous medium and photonic crystal left-handed materials

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Results of the delay time for the electromagnetic wave to reach its final direction through both photonic crystal and homogeneous medium are presented. The delay- or transient-time results, based on different cases and different incoming wave angles, show that the diffracted beam is trapped at the interface. This delay time is longer for the negative refractive index photonic crystal and is almost twice the duration of the delay time for the positive one. For the homogeneous medium, we also find that at the interface between a right- and left-handed medium the delay time is even longer than in the photonic crystal case. A comparison between left-handed behavior in photonic crystals and homogeneous media is reported.
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Veselago[1] predicted that materials with simultaneously negative permittivity ϵ and permeability μ have a wave vector **k**, electric field **E**, and magnetic field **H**, forming a left-handed set of vectors. As a result, these materials exhibit many unusual properties, such as negative index of refraction, $n=-\sqrt{\epsilon\mu}$, antiparallel wave vector **k**, and Poynting vector (**S**) antiparallel phase (v_p), and group (v_g) velocities, etc. An electromagnetic (EM) wave incident on the interface of such a medium will refract the "wrong way," i.e., negatively. Thus, materials with these properties are called left-handed materials (LHM), or negative index materials (NIM).[1–3] Recent experimental and theoretical studies confirmed the reality of negative refraction.[4–7] In fact, composite metallic media consisting of split ring resonators (SRR) and wires,[2,3,5,8] as well as photonic crystals (PCs),[4,9–11] have negatively refractive properties under certain conditions. These systems still attract great attention today due to their vast potential in applications, such as in superlensing.[12,13]

In Ref. 4 it was shown that the negative refraction at the PC interface is not an instantaneous phenomenon, and that the EM wave takes time to reach its final direction. In this letter we report numerical simulation results that compare delay-time phenomena between positive and negative refractive index media and provide a plausible explanation for the larger delay times present in the negative index media. For this purpose we study two kinds of negatively refracting systems: the photonic crystal and the homogeneous dispersive medium.

The photonic crystal, a periodic arrangement of elements with a positive index of refraction, is an inherently lossless system. It was found that such a system has an effective index controllable by the band structure.[4,9,11] In our study we focus on cases with "almost" isotropic dispersion, where the PC resembles in many respects a homogeneous medium with a positive or negative refractive index.[4] The PC system we consider here is a two-dimensional hexagonal lattice of cylindrical rods with dielectric constant 12.96 and radius $r=0.35a$. We employ the finite-difference-time domain (FDTD) technique with perfect matched layer (PML) boundary conditions[14] and study the time and space evolution of the incident beam as it refracts at the interface and propagates inside the PC system. The EM wave source has a Gaussian profile in space and is placed outside of the structure at various angles. The source emits an "almost" monochromatic wave, at the frequency of interest, and with a magnetic field perpendicular to the plane of incidence [H-(TE) polarization].

We study this PC system for cases with a positive and negative refractive index, at different frequencies, and for many incident angles. In particular, for frequencies $\tilde{f}=0.48$ and 0.49, the PC has a positive refractive index ($n=0.43$ and 0.59, respectively). For frequencies $\tilde{f}=0.58$ and 0.59, the PC has a negative refractive index ($n=-0.69$ and -0.55, respectively). Note that $\tilde{f}=\omega a/2\pi c$ is the frequency in dimensionless units, where ω is the frequency, a is the lattice constant, and c is the velocity of light. Figure 1 displays the magnetic field of the incoming, reflected and refracted beam at the interface between vacuum ($n=1$) and the PC (cut along the ΓK direction). In Fig. 1(a) we see a case with a positive refractive index, while in Fig. 1(b) we see a case with a negative refractive index. These snapshots depict the field after a long time has passed and the refracted beam propagates in its final direction. Nonetheless, a corresponding time sequence of the beam propagation shows that the beam does not refract instantaneously to the positive [Fig. 1(a)] or to the negative direction [Fig. 1(b)]. We initially observe a trapping of the wave at the interface. Afterwards, the wave reorganizes and propagates towards its final direction. These effects are more prominent for the cases with a negative refractive index. From these types of time sequences for all the aforementioned cases, we make a rough estimate for the transient time in units of the period (T). We summarize our results in Table I. When we refer to transient time, we mean the time the refractive beam needs to reach its final propagation direction. We see that in the negative refractive index PC the transient time is larger by approximately a factor of 2. Incidentally, the group velocity of the EM wave in the negative index PC cases is similar, if not larger than, in the positive ones.[15] Thus, the group velocity does not account for the

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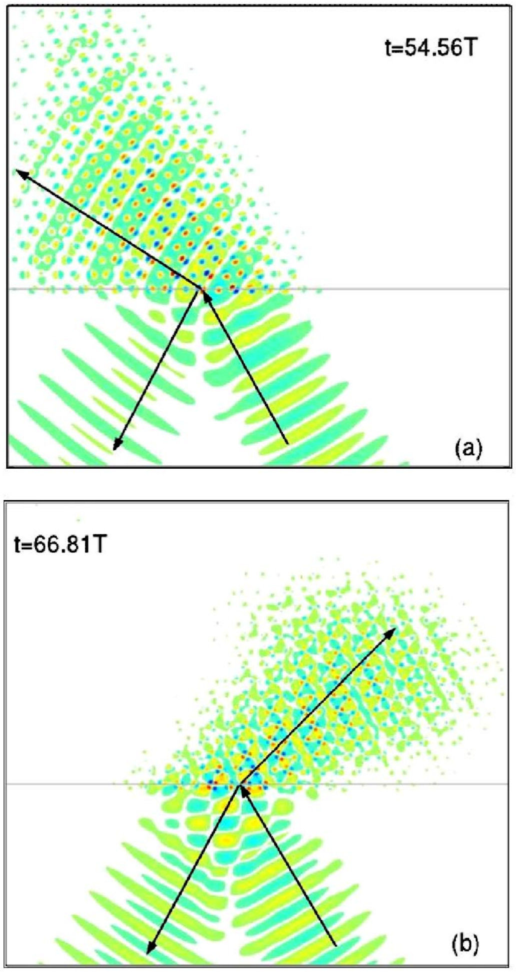

FIG. 1. (Color online) The magnetic field of the Gaussian beam undergoing reflection and refraction for (a) positive refractive index PC with $n=0.59$ for $\tilde{f}=0.49$, and (b) negative refractive index PC with $n=-0.69$ for $\tilde{f}=0.58$. T is the period $2\pi/\omega$.

large difference in the observed transient times between the positive and negative index PC systems. Therefore, our results indicate a correlation between large transient times and negative refractive index. A possible explanation for the long time delay for the negative n case is that the component of the wave vector $\mathbf{k}$ that is perpendicular to the interface, $k_{\perp}$, reverses direction when entering the negative n region. We have observed a slightly larger transient time for smaller angles of incidence for the negative index PC medium (see Table I). For the smaller angles, the magnitude for the change of $k_{\perp}$ is slightly larger.

To factor out any effects that may be present due to the periodicity of the PC system, we study a homogeneous dispersive medium. We implement a dispersive model for both permittivity ϵ and permeability μ:

TABLE I. Transient time for various angles and different frequencies. Notice that the transient time for $\tilde{f}=0.48$, in the last column, corresponds to 23° and not to 30°.

$\tilde{f}$	n	8°	15°	30°
0.58	−0.69	33–43 T	34–41 T	27–41 T
0.59	−0.55	32–38 T	30–35 T	28–36T
0.48	0.43	13–16 T	12–16 T	13–17 T
0.49	0.59	12–16T	11–14T	13–17T

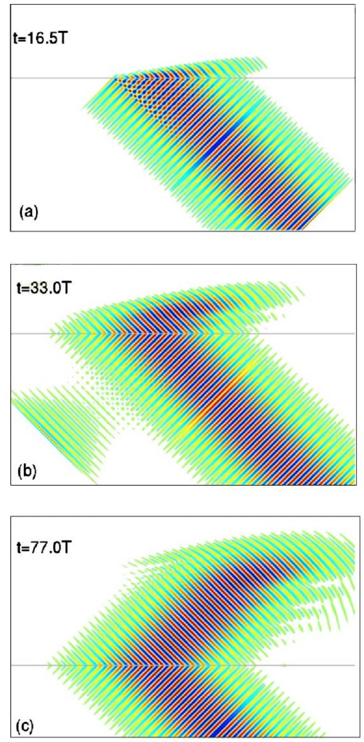

FIG. 2. (Color online) The magnetic field of the Gaussian beam undergoing reflection and refraction at the interface of a homogeneous dispersive medium with $n=-1$. T is the period of the incident EM wave.

$$\varepsilon = \varepsilon_0\varepsilon_r\left(1-\frac{\omega_p^2}{\omega^2}\right), \quad \mu = \mu_0\mu_r\left(1-\frac{\omega_p^2}{\omega^2}\right). \tag{1}$$

ω_p is the plasma frequency, ε_0 and μ_0 are the vacuum permittivity and permeability, and ε_r and μ_r are the relative permittivity and the permeability. In each of the subsequent cases, the parameters ε_r, μ_r, and ω_p are chosen appropriately to obtain the desired values for ϵ and μ. Note that the refractive index is negative below the plasma frequency and has exactly the value of −1 when $\omega=\omega_p/\sqrt{2}$.

To study the refraction at the interface of the dispersive medium, we implement the FDTD technique for a dispersive system in a manner analogous to the one in Ref. 16. We place the source (Gaussian in space and "almost" monochromatic) at an angle of 45°. The incident wave has wavelength $\lambda=10$ mm and magnetic field perpendicular to the plane of incidence (H polarization). A time step of $\delta t=0.185$ ps and a spatial grid with $dx=dy=0.098$ mm are considered. We focus on two cases, one with negative refractive index $n=-1$ and one with positive refractive index $n=0.8$.[17] In Fig. 2 we show the time sequence of the EM wave propagation inside the negative refractive index medium with $n=-1$. Since $n=-1$, we expect the beam to refract negatively at an angle of −45°. However, at early times [Fig. 2(a)] the refracted beam seems to propagate parallel to the interface—a similar phenomenon we observed for the negative index PC. After an elapsed time of 33 periods, the angle of the refracted beam is about $\theta=-20°$ [Fig. 2(b)]. The angle of the refracted beam approaches the predicted value only after a time of 77 periods. For the medium with $n=0.8$, the refracted beam approaches the predicted propagation angle $(\sim 62°)$ after 30 periods. So, the transient time in the negative index homogeneous medium is longer than in the positive one. This transient time involves both the delay at the interface and the

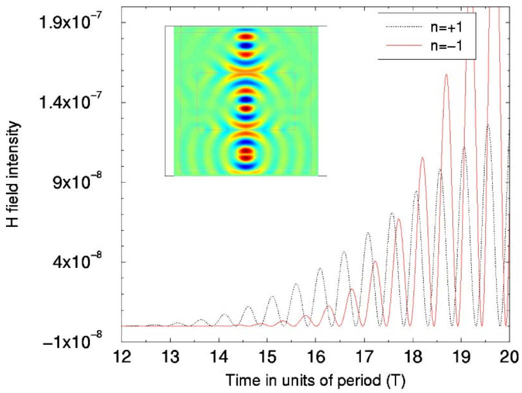

FIG. 3. (Color online) The magnetic-field intensity vs time for $n=1$ (solid line) and $n=-1$ (dotted line), at a point located at 1.875λ away from a homogeneous dispersive slab. A source with λ=10 mm located at 1.5λ before the slab is considered. (Inset.) The magnetic field inside the structure with $n=-1$ after a time of 22 periods is shown.

time it takes for the beam to rearrange into the final propagation direction.

To investigate the delay time at the interface, we calculate the time it takes an EM wave to propagate through a finite slab of material. In particular, we consider the source placed at 1.5λ in front of the first interface of the slab. We monitor the field at a point 1.875λ away from the second interface. The thickness of the homogeneous slab is 3.5λ. We study two cases, with $n=1$ and −1. In both cases, the incoming wave has a wavelength λ=10 mm and a group velocity v_g inside the slab equal to $c/3$.[18] In Fig. 3 we show the intensity of the field at the specified point. The dotted line represents the case with refractive index $n=1$, while the solid line represents the case with $n=-1$. It is clear that the beam propagating through the negative index slab is delayed by about 2.5 T. Evidently, the delay time at the interface is associated with a negative refractive index. Thus, the existence of dispersion itself can neither explain the large transient time in the cases of negative refractive index, nor the delay time observed in Fig. 3.

In conclusion, we systematically studied transient-time phenomena associated with refraction of an EM wave at the interface of materials with a positive or negative refractive index. In negative index media (both photonic crystal and homogeneous) larger transient times were observed. In addition, we found that the delay time at the interface is longer for the homogeneous medium with a negative index than for the medium with a positive index. This delay is not due to the dispersion of the medium. This time delay is most probably due to the change of the direction of $k_\perp$, as the EM wave enters the negative n region.

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