Absorption losses in periodic arrays of thin metallic wires

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We analyze the transmission and reflection of electromagnetic waves calculated from transfer matrix simulations of periodic arrangements of thin metallic wires. The effective permittivity and the absorption of the arrangements of wires are determined. Their dependence on the wire thickness and the conductance of the metallic wires is studied. The cutoff frequency, or effective plasma frequency, is obtained and compared with analytical predictions. It is shown that the periodic arrangement of wires exhibits a frequency region in which the real part of the permittivity is negative while its imaginary part is very small. This behavior is seen for wires with thickness as small as 17 μm with a lattice constant of 3.33 mm.

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Rapidly increasing interest in left-handed metamaterials (LHMs) (for a recent review, see Ref. 1) has raised some interesting questions about the electromagnetic (EM) properties of composites that contain thin metallic components. The simplest example of such a composite is a periodic array of thin metallic wires. Pendry *et al.*[2] predicted that such a system behaves as a high-pass filter with an effective permittivity

$$\epsilon_{\text{eff}} = 1 - \frac{{f_p}^2}{f^2 + 2i\gamma f}\,. \tag{1}$$

In Eq. (1), f_p is the effective plasma frequency, or cutoff frequency,[3] and γ is the damping factor. Various theoretical formulas were derived for the dependence of the plasma frequency on lattice period a and wire radius r. Pendry *et al.*[2] found that

$${f_p}^2 = \frac{{c_{\text{light}}}^2}{2\pi a^2 \ln(a/r)}\,, \tag{2}$$

Sarychev and Shalaev[4] found that

$${f_p}^2 = \frac{{c_{\text{light}}}^2}{2\pi a^2[\ln(a/\sqrt{2}\,r) + \pi/2 - 3]}\,, \tag{3}$$

and Maslovski *et al.*[5] found that

$${f_p}^2 = \frac{{c_{\text{light}}}^2}{2\pi a^2[\ln a^2/4r(a-r)]}\,. \tag{4}$$

In Eqs. (2)–(4), c_{light} is the velocity of light in vacuum.

Periodic arrangements of thin metallic wires are used as a negative-ϵ medium[6,7] in the left-handed structures.[8–10] It is therefore important to understand how the EM response—not only the effective plasma frequency but also the factor γ—depends on the structural parameters of the wire system. Recently, Ponizhovskaya *et al.*[11] claimed that for a small wire radius the absorption in the wire system is so large that the transmission losses do not allow any propagation of EM wave in the left-handed structure. Very low transmission, measured in the original experiments on LHM,[8,9] seemed to agree with the pessimistic conclusion of Ponizhovskaya *et al.* However, it is not clear why the transmission was so low in the original experiments. Recent experimental measurements[10,12] established that the transmission of LHM could be as good as in right-handed systems.

Our aim in this Letter is to study numerically how the effective permittivity of the periodic arrangement of metallic wires depends on the wire radius and on the conductance of the wires. We present results for the real (ϵ_{eff}') and imaginary (ϵ_{eff}'') parts of the effective permittivity of the wire medium, estimate the transmission losses and the plasma frequency, and compare our results with the analytical formulas given in Eqs. (1)–(4).

In our numerical simulations we use the transfer matrix method (TMM). Details of the method are given elsewhere.[13] Here we point out only the main advantage of the TMM, namely, that it gives directly the transmission, t, reflection, r, and absorption, $A = 1 - |t|^2 - |r|^2$, of the EM plane wave passing through the system. However, in the finite-difference time-domain method, which was used in the study reported in Ref. 11, one obtains t and r from the time development of the wave packet, which is a much more complicated and probably also less accurate calculation. To be able to obtain ϵ_{eff}, one also needs the phases of r and t, in addition to their amplitudes. This result is also easily achieved with the TMM.

From the obtained data on transmission and reflection, we calculate the effective permittivity of the system.[14] The refractive index is given by

$$\cos(nkL) = \frac{1}{2t}(1 - r^2 + t^2)\,. \tag{5}$$

Since we do not expect any magnetic response, we fixed the value of the permeability to be $\mu \equiv 1$. The permittivity is then found as $\epsilon_{\text{eff}} = n^2$.

The way that we discretize the space in the TMM might control the accuracy of our results. To test how discretization influences our results, we repeated the numerical simulation for different discretizations.

The wire is represented as a rectangle with a square cross section $2r \times 2r$, r is the wire radius. The obtained results for the effective permittivity ϵ_{eff} (both real and imaginary) are almost independent of the discretization procedure.

Figure 1 shows how the effective permittivity depends on the wire radius. We analyzed four different wire arrays with period $a = 3.33$ mm. For all of them the real part of the effective permittivity is negative and can be fitted by Eq. (1). This enables us to obtain easily the plasma frequency. Only for the smallest wire thickness studied ($17\ \mu\text{m} \times 17\ \mu\text{m}$) does one get relatively large ϵ_{eff}'' for small frequencies. In this region, Eq. (1) is not valid. Nevertheless, for frequencies larger than 5 GHz, ϵ_{eff}'' is small and ϵ_{eff}' is negative.

The bottom right-hand plot in Fig. 1 shows data for wires with a cross section of $17\ \mu\text{m} \times 300\ \mu\text{m}$. These parameters were used in the experiment of Shelby *et al.*[9] We again see that the result given by Eq. (1) agrees qualitatively with our data. Thus, there is no doubt that this array of wires really produces a medium with negative ϵ_{eff}', which then can be used in the creation of left-handed systems.

Figure 2 compares our data for plasma frequency with the analytical formulas given by Eqs. (2)–(4). Accepting some uncertainty in the estimation of the plasma frequency from the numerical data, we can conclude that for thin wires our data agree with theoretical formula (3) of Sarychev and Shalaev.[4] For thicker wires, our results are in agreement with formula (4) of Maslovski *et al.*[5]

We also study how the effective permittivity depends on the conductance of the metallic wires. In the simulations shown in Fig. 1, we consider the metallic permittivity to be $\epsilon_m = (-3 + 588\, i) \times 10^3$. We are aware that this value of ϵ_m is smaller than the permittivity of realistic metallic wires: For instance, for copper, $\epsilon_m \approx 5 \times 10^7\, i$, as follows from the relation between the permittivity and the conductance[15] [the conductivity of copper is $\sigma \approx 5.9 \times 10^7 (\Omega \text{m})^{-1}$]. Our data in Fig. 1 therefore underestimate losses, because transmission losses are smaller for higher values of ϵ_m.[16] This result is clearly shown in Fig. 3, in which we present the ratio $\kappa = |\epsilon_{\text{eff}}''/\epsilon_{\text{eff}}'|$ versus frequency for two systems that differ only in the value of the imaginary part of ϵ_m. Figure 3 also shows the frequency dependence of the absorption as obtained from the numerical simulations. The absorption also exhibits a maximum in the neighborhood of the plasma frequency.

Figure 1 also confirms that ϵ_{eff}'' increases when the wire radius decreases. For instance, γ is only 0.003 GHz for $r = 100\ \mu$m. As we show in Fig. 4, γ increases to 1.2 GHz when the wire radius decreases to 15 μm. Nevertheless, even for wires of thickness $17\ \mu\text{m} \times 17\ \mu\text{m}$ losses are much less than what was claimed in Ref. 11. As shown in Fig. 1, an array of wires with thickness $17\ \mu\text{m} \times 17\ \mu\text{m}$ also creates a negative-ϵ medium. As this result is in strong contrast with the results of Ref. 11, we decided to study exactly the same system as that of Ref. 11. The results of our simulations are shown in Fig. 4. Although such systems are not used in experimental arrangements of LHM, our results give a comparison between two different numerical treatments. Our

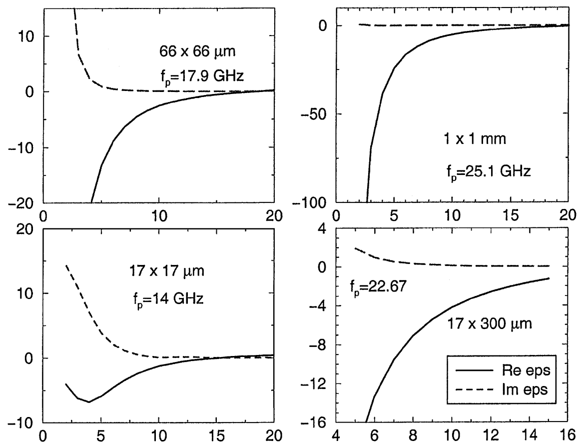

Fig. 1. Effective permittivity as a function of frequency for various shapes of the metallic wires. The lattice period in all cases is $a = 3.33$ mm. We used metallic permittivity $\epsilon_m = (-3 + 588i) \times 10^3$.

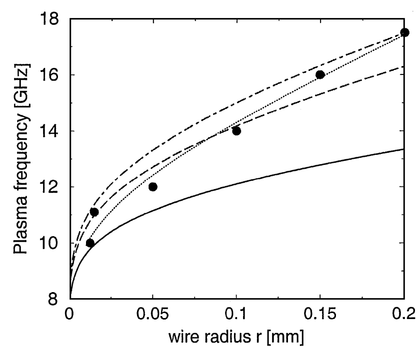

Fig. 2. Plasma frequency as a function of the wire radius. The lattice constant is $a = 5$ mm. The solid, dashed, and dotted–dashed curves are the results from Pendry *et al.*,[2] Sarychev and Shalaev,[4] and Maslovski *et al.*,[5] respectively. The dotted curve is a fit of our data to the function $f_p = a_0/\sqrt{\ln(a_1/r)}$ with parameters $a_0 = 20.9$ and $a_1 = 0.84$.

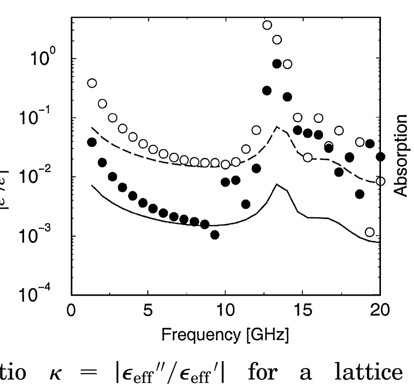

Fig. 3. Ratio $\kappa = |\epsilon_{\text{eff}}''/\epsilon_{\text{eff}}'|$ for a lattice of wires with radius 50 μm. The metallic permittivity is (open circles) $\epsilon_m = (-3 + 588i) \times 10^3$ and (full circles) $\epsilon_m = (-3 + 5\,880i) \times 10^3$. The dashed (solid) curve is absorption for the corresponding system obtained numerically by the TMM. These numerical results confirm that losses are smaller for higher metallic permittivity and that the value of the plasma frequency, estimated approximately from the position of the maximum of κ, does not depend on the value of the metallic permittivity.

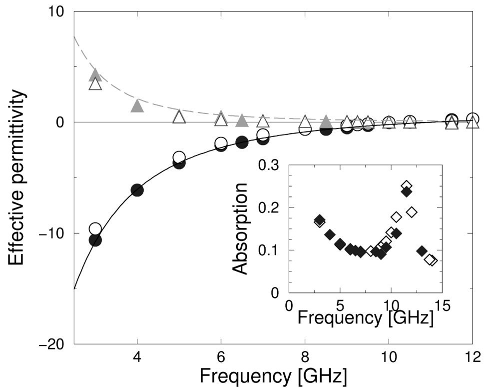

Fig. 4. Effective permittivity for a lattice of thin metallic wires. The wire radius is $r = 15\ \mu$m, and the lattice constant is $a = 5$ mm. The metallic permittivity is $\epsilon_m = -2000 + 10^6\, i$. Two different discretizations are used with mesh sizes of (open symbols) 30 μm and (filled symbols) 15 μm. The solid and dashed curves are fitted to Eq. (1) with $f_p = 11.1$ GHz and $\gamma = 1.2$ GHz. The length of the system was (open symbols) up to 60 unit lengths and (filled symbols) 10 unit lengths. The inset shows the numerically calculated absorption as a function of frequency.

data again clearly show that ϵ_{eff}' is negative for $f < f_p$. As shown in the inset of Fig. 4, the transmission losses are also small.

We believe that the present data are more accurate than those published in Ref. 11, not only because they agree with the theoretical analysis but also because the TMM gives the reflection and its phase straightforwardly. Exact estimation of the reflection is important because the main difference between our results and those of Ref. 11 seems to be in the estimation of reflection $R = |r|^2$. When we compare our data for absorption, given in the inset of Fig. 4, with those given in Fig. 2b of Ref. 11 we see that our absorption is much less than that estimated in Refs. 9, 10, and 12.

In conclusion, we have analyzed numerically the transmission properties of a periodic arrangements of thin metallic wires. From the transmission and reflection data we calculate the effective permittivity and plasma frequency, which agree qualitatively with theoretical predictions. Both the effective permittivity and the absorption data confirm that the array of thin metallic wires used in recent experiments on the left-handed metamaterials[11] indeed behaves as a negative-permittivity medium with low losses.

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