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The probability distribution of the conductance at the mobility edge

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Abstract

The probability distribution of the conductance $p(g)$ of disordered 2d and 3d systems is calculated by transfer matrix techniques. As expected, $p(g)$ is Gaussian for extended states while for localized states it is log–normal. We find that at the mobility edge $p(g)$ is highly asymmetric and universal.

Keywords: Conductance; Mobility edge; Localization; Metal–insulator transition; Disorder

In the presence of disorder [1], a system may undergo a transition from insulating to metallic behaviour as the Fermi energy varies in an energy range containing both localized and extended states, seperated by a mobility edge. This transition can be characterized either by transport properties like, e.g. the conductance, or by properties of the eigenstates of the system like, e.g. the correlation length ξ_c (approaching from the metallic side of the transition) or the localization length ξ_ℓ (approaching from the insulating side of the transition). While the latter are self–averaging quantities, i.e. the ensemble average may be used as a scaling variable, the conductance is not [2–7]. Therefore, it is of great importance to determine the complete probability distribution $p(g)$ of the conductance g (in units of e^2/h), especially at the critical point of the metal–insulator transition, as it is well known to be a Gaussian on the metallic side and log–normal on the insulating side. The correct form of $p_c(g)$, the distribution of the conductance at the mobility edge [2–7], is still not sufficiently well-known.

Using a tight–binding model [8] with diagonal disorder, a transition from a metallic state to an insulating one can be induced [9] in a finite-size sample by increasing the disorder strength W. In all our results, W is given in units of the hopping integral. The localization length ξ_ℓ decreases as the strength of the disorder, W, increases. As long as ξ_ℓ is much bigger than the system size, the electron will cross the sample with ease, thus being essentially delocalized. If, on the other hand, ξ_ℓ is sufficiently smaller than the system size, the electron will become localized in a small region and not contribute much to the conductance. The critical strength of disorder W_c will occur where the localization length becomes comparable to the system size. We have systematically studied the conductance g of the 2d and 3d tight–binding model by using the transfer matrix technique, which relates

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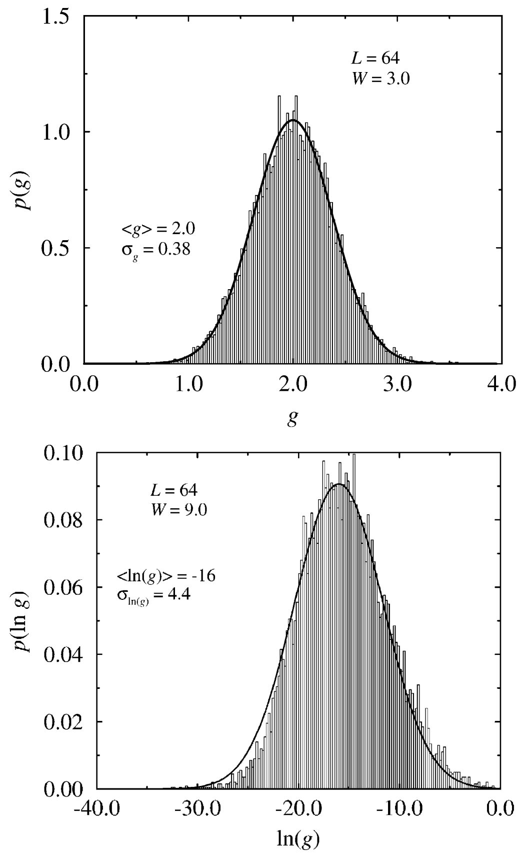

Fig. 1. The distribution of the conductance for a square of 64×64 lattice sites. Small disorder ($W = 3$, top panel) leads to metallic behaviour; strong disorder ($W = 9$, bottom panel) leads to insulating behaviour. The values for the average and standard deviation of the Gaussian fits are given in the graphs.

g to the transmission matrix $\boldsymbol{t}$ by $g = 2\,\mathrm{Tr}(\boldsymbol{t}^{\dagger}\boldsymbol{t})$. The g defined here is for both spin orientations. Fig. 1 shows the distributions of the conductance g and the natural logarithm of the conductance $\ln g$ for a metallic and a localized 2d sample, respectively. In the "metallic" regime (it is really weakly localized, since we are working in 2d) where $W = 3.0$ we have $\xi_{\ell} = 47234$, which is much larger than the system size $L = 64$. In the localized regime, where $W = 9.0$, we have $\xi_{\ell} = 7.54$, which is smaller than the size of the system. Both distributions can be fitted very well by a Gaussian normal distribution [7]. Fig. 2 displays the distributions for a system near the critical point. The distribution of $\ln g$

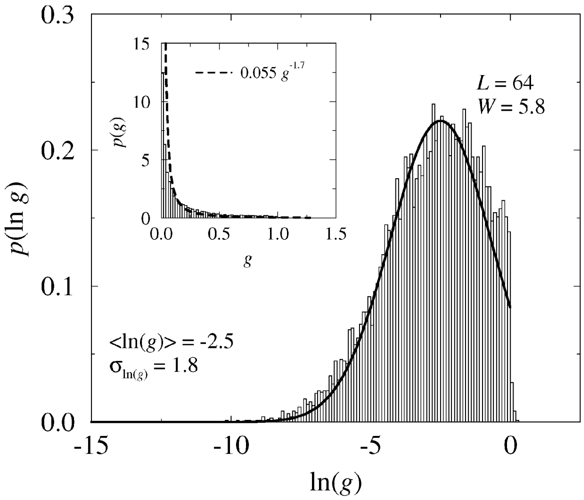

Fig. 2. The distribution of the conductance for a square system of 64×64 lattice sites. The disorder was chosen such that the localization length is close to the system size L.

shows a characteristic cut-off at $\ln g = 0$. Only a few samples have a conductance $g > 1$. This is in agreement with analytical results [5].

Applying a strong magnetic field perpendicular to a 2d system creates states with a diverging localization length [6] even in the thermodynamical limit, as long as the disorder is not too strong. Thus, one can approach a critical state in such a system by varying the energy, even though one cannot reach a true metallic state. In Fig. 3, we have an insulating system, the conductance distribution again fitting well to a log-normal distribution. The distribution for the critical state $p_c(g)$, shown in Fig. 4, again has the abrupt cut-off at $\ln g = 0$. It can be fitted to a skewed log-normal distribution, which is normalized on the interval $(-\infty; 0]$ rather than all real numbers. Notice that in this case also $p_c(g)$ is highly asymmetric and very similar to the case shown in Fig. 2.

Comparing these results to a 3d system without magnetic field, again varying the disorder strength and keeping the energy fixed at $E = 0.0$, we find the same qualitative picture: the distribution of the conductance is normal on the metallic side and log-normal on the insulating side of the transition (see Fig. 5), whereas the critical state is

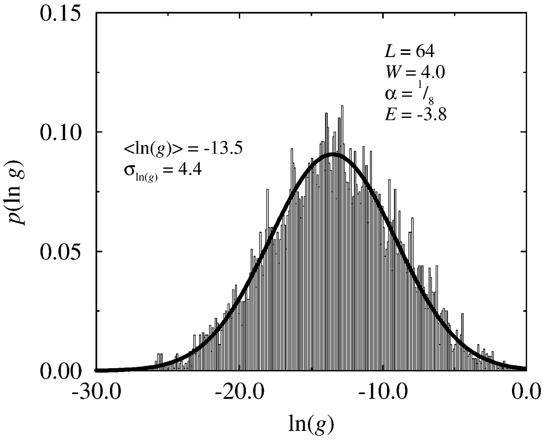

Fig. 3. The conductance distribution of a square system of 64×64 lattice sites. For the chosen disorder strength $W = 4$ and magnetic flux $\alpha = \frac{1}{8}$ states at the energy $E = -3.8$ are well localized. The parameters for the Gaussian fit are given in the graph.

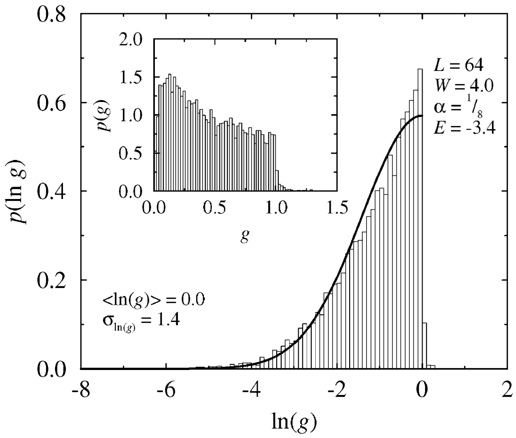

Fig. 4. The conductance distribution of a square system of 64×64 lattice sites. For the chosen disorder strength $W = 4$ and magnetic flux $\alpha = \frac{1}{8}$ states at the energy $E = -3.4$ are critical. The parameters for the fit are given in the graph.

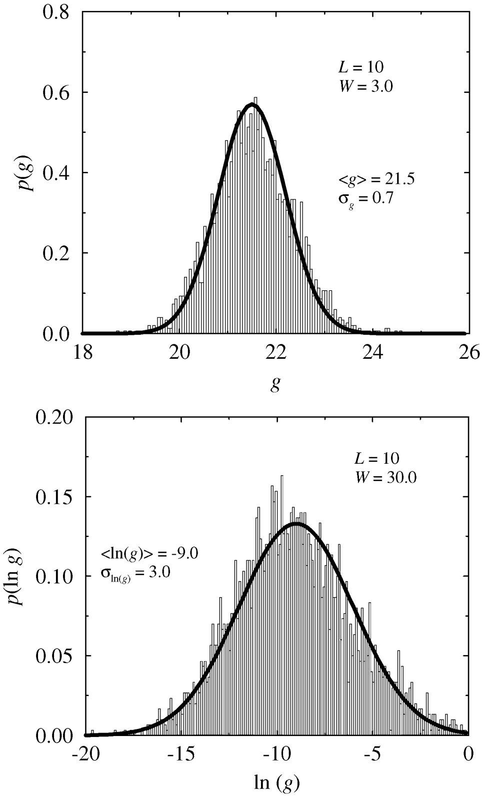

Fig. 5. The conductance distribution of a cubic system of $10 \times 10 \times 10$ lattice sites. Small disorder ($W = 3$, left panel) leads to metallic behaviour; strong disorder ($W = 30$, right panel) leads to insulating behaviour. The values for the average and standard deviation of the Gaussian fits are given in the graphs.

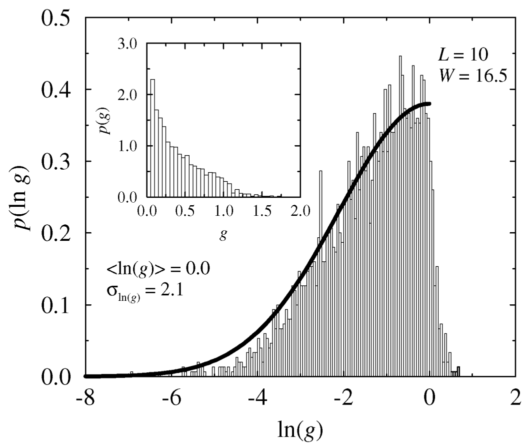

Fig. 6. The conductance distribution of a cubic system of $10 \times 10 \times 10$ lattice sites. The disorder was chosen such that the localization length is close to the system size L. The parameters for the fit are given in the graph.

characterized by a cut-off at $\ln g = 0$ and a skewed log-normal distribution for $\ln g \leqslant 0$ (see Fig. 6).

In conclusion, our detailed numerical results show that the probability distribution of the conductance is normal for the extended regime and log-normal for the localized regime. However, at the mobility edge $p(g)$ is highly asymmetric. The form of $p_c(g)$ at the critical point is independent of the dimensionality of the system and of the model. This suggests that $p_c(g)$ is universal.

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