What is the Right Form of the Probability Distribution of the Conductance at the Mobility Edge?

In a recent Letter, Slevin and Ohtsuki [1] reported finite size scaling results for the Anderson metal-to-insulator transition for the orthogonal and unitarity classes of the single electron tight-binding (TB) model. The average value of the conductance \( G = (e^2/h)g \) at the mobility edge, as well as the distribution of the conductance at the critical point \( p_c(g) \), was calculated. Their studies showed that \( p_c(g) \) is independent of the system size. It also does not show any dip around \( g = 0 \), as the \( \epsilon \) expansion results [2] suggest. We will present new numerical data that indeed shows that \( p_c(g) \) has a dip for small \( g \).

We have systematically studied the conductance \( G \) of the 3D TB model by using the transfer matrix technique [3], which relates the conductance \( G \) with the transmission matrix \( t \), i.e., \( G = (e^2/h)g \), with \( g = 2 \mathrm{Tr}(tt^\dagger) \). The \( g \) defined here is for both spins. In Fig. 1 we present the results of \( p_c(g) \) for three different sizes of \( N = 5, 10, \) and 20.

The mobility edge [1] is at \( W = 16.5 \) and \( E = 0.0 \). Notice that as the size of the system increases a dip is developed at \( g = 0 \), which is not present in the results presented in Fig. 2 of Ref. [1]. We therefore have a size dependent \( p_c(g) \), which has a dip at small \( g \). \( p_c(g) \) for extended states is Gaussian, while for localized states it is log normal [4]. However, it is not well known either experimentally [5] or theoretically what is the correct form of the probability distribution at the mobility edge. \( p_c(g) \) obtained [2] in the \( \epsilon \) expansion in the field theory has a hole at small \( g \), in agreement with the numerical results presented here. Recent results [6] for a 2D TB model in the presence of a strong magnetic filed show that \( p_c(g) \) is very broad with a dip at small \( g \). The 2D \( p_c(g) \) is very different from the 3D \( p_c(g) \) presented here. We have also calculated the average value of the conductance at the critical point \( (E = 0.0 \) and \( W = 16.5) \) for \( N = 5, 10, \) and 20 for 20000, 10000, and 8000 random configurations, respectively. The results are summarized in Table I.

Notice that both \( g \) and \( \ln g \) have very large standard deviations, as big as their average values. Our results for both \( \langle g \rangle \) and \( \langle g \rangle_g = e^{\langle \ln g \rangle} \) for the \( N = 10 \) case \((0.78, 0.48)\) are larger than the results presented \((0.58, 0.30)\) in Table III of Ref. [1] for the same model. This difference might be due [7] to the different boundary conditions used by Ref. [1] (fixed) and ourselves (periodic). For the 2D case [6], it is shown that \( \langle g \rangle = 1.00 \) and \( \langle g \rangle_g = 0.88 \) for the infinite size system. If we extrapolate our finite size results to infinite sizes we obtain that \( \langle g \rangle = 1.00 \) and \( \langle g \rangle_g = 0.70 \). Remember that \( \sigma_g \) is comparable to \( \langle g \rangle \).

In summary, the critical conductance distribution \( p_c(g) \) in the orthogonal case does not increase without limit as \( g \to 0 \), contrary to the impression given by the Letter of Slevin and Ohtsuki [1]. We present evidence, based on direct numerical calculation of the conductance via the Landauer formula, that the critical conductance distribution does indeed tend to zero, in agreement with the \( \epsilon \) expansion results [2].

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FIG. 1. The distribution of \( g \) at the critical point for three different sizes.