Wave propagation in nonlinear multilayer structures

Elefterios Lidorikis, Qiming Li, and Costas M. Soukoulis
Ames Laboratory–USDOE and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011
(Received 28 June 1996)

We investigate the adequacy of the Kronig-Penney δ-function model in describing the electromagnetic wave propagation in periodic structures consisting of thin layers of materials with an intensity-dependent dielectric constant. We find that the model captures the most essential features of nonlinear response to radiation. Excellent agreement is found between the results from the δ-function model and the exact solutions of nonlinear wave equations. However, discrepancies do exist below the bottom of transmission bands due to the rigidness of the band edge in the δ-function model. Consequently, gap solitons cannot form in the δ-function model when the nonlinear Kerr coefficient is positive. [S0163-1829(96)03740-X]

The presence of nonlinearity is known to lead to a much richer and more complex optical response to radiation. One such phenomenon, known as bistability, in which two possible output states exist for a single input, promises important potential applications in ultrafast optical switches.[1] A bistable response can be observed in simple structures such as the traditional nonlinear Fabry-Perot etlons[2] or in distributed feedback structures such as multilayered systems consisting of alternating nonlinear dielectric materials.[3] In essence, a bistable response results from the modulation of transmission by an intensity-dependent phase shift. The multilayer structure can also act as a Bragg reflector and offers additional transmission modes known as gap solitons within the stop band.[4] The optimal coupling of these localized modes to radiation may lead to switching thresholds orders of magnitude smaller than achievable in the transmission band.[5]

Recently, global transmission diagrams of multilayer structures with a Kerr nonlinearity were investigated within a Kronig-Penney δ-function model.[6] It was found that the effectiveness of the nonlinearity is strongly modified by the frequency. In addition, the nonlinear responses of the positive and negative nonlinear media are distinctly different due to the modulation of the dispersion relation by the superlattice. Many dominant features were understood through an analysis of stable periodic orbits of the corresponding nonlinear mapping, as well as an analysis of various spectrum and stability bounds of the nonlinear difference and the corresponding differential equations. A simple and intuitive picture of the formation of gap solitons and soliton trains, based on a mechanical analogy, was also presented. These understandings may prove useful for incorporating nonlinearity in systems of higher dimensions, for example, in photonic band-gap structures.[7]

All these results were based on the Kronig-Penney δ-function model. Such a model offers the advantage of being amenable to some analytical treatment, and is expected to work well when the nonlinear layer is thin compared with the wavelength of the incident wave. However, to our knowledge, its general adequacy as well as its limitations have not been previously investigated. In this Brief Report, we compare results derived from the Kronig-Penney δ-function model with direct numerical solutions of wave propagation in nonlinear superlattices of finite thickness. We find the Kronig-Penney δ-function model captures most of the essential features of the nonlinear response in superlattice structures. The global transmission diagrams from the two methods are in excellent agreement with each other. However, some disagreement does exist, most significantly below the bottom of the transmission band. This difference is entirely due to the rigidness of the band edge in the Kronig-Penney δ-function model, a unphysical feature that affects the conclusion regarding the existence of gap solitons when the Kerr nonlinearity is positive.

The formulation of the steady state plane wave transmission problem in nonlinear superlattices has been described in detail elsewhere.[6] The structure consists of alternating layers of two dielectric materials, one of which has an intensity-dependent Kerr nonlinearity, $\epsilon=\epsilon_0+\alpha_2|E|^2$. For normal incidence of a plane wave, the electric-field amplitude $E(x)$ satisfies the equation

$$\frac{d^2E(x)}{dx^2}+\frac{\omega^2}{c^2}\epsilon(x)E(x)=0, \tag{1}$$

where ω is the optical frequency, and c is the vacuum speed of the light. $\epsilon(x)$ is the dielectric constant which varies along the structure, and depends on the local-field intensity at nonlinear layers. The transmission characteristics are obtained by solving Eq. (1) under the boundary condition

$$E(x,t)=\begin{cases} E_0e^{i(kx-\omega t)}+E_re^{-i(kx+\omega t)} & \text{for } x<0 \\ E_te^{i(kx-\omega t)} & \text{for } x>L, \end{cases} \tag{2}$$

where E_0, E_t, and E_r are the amplitude of the incident, transmitted, and reflected waves, respectively. Wave vector $k=\omega/c$, and L is the total length of the structure. The transmission coefficient T is defined as $T=|E_t|^2/|E_0|^2$.

Equation (1) can be solved[4] by matching analytical solutions in each nonlinear layer at the layer interfaces, which may be expressed in terms of the Jacobi elliptic functions.[4] A much simpler numerical approach, however, is first to discretize the structure and then iterate the difference equation numerically across the sample, starting from the output field E_t. Our numerical results are obtained this way.

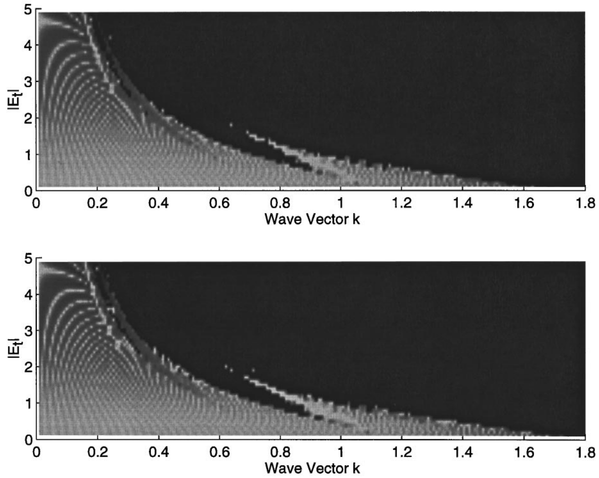

FIG. 1. The transmission diagram for a nonlinear superlattice of $L=80$ units with a positive Kerr coefficient. (a) δ-function model with $\alpha=1$, and (b) exact solutions with linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=10$. Higher bands (not shown) show similar behavior.

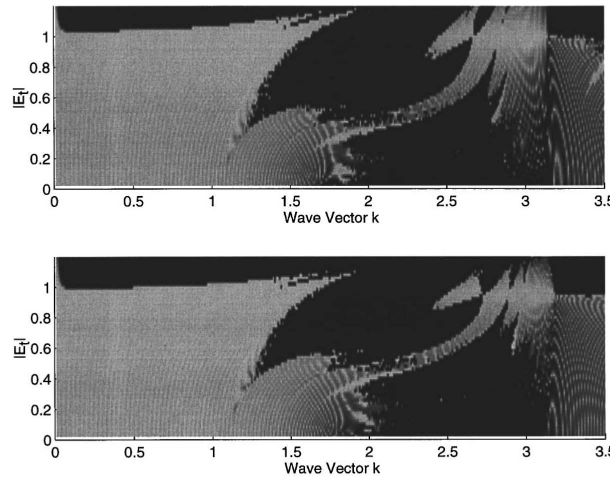

FIG. 2. The transmission diagram for a nonlinear superlattice of $L=80$ units with a negative Kerr coefficient. (a) δ-function model with $\alpha=1$. (b) Linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=10$.

The Kronig-Penney δ-function model, on the other hand, describes a system with infinitesimally thin nonlinear layers.[6] In this model, the electric field obeys

$$\frac{d^2E(x)}{dx^2}+\frac{\alpha\omega^2}{c^2}\sum_{n=1}^{N}[1+\lambda|E(x)|^2]E(x)\,\delta(x-n)+k^2E(x)=0. \tag{3}$$

This can be easily rewritten[6] as a difference equation in terms of the field at the nonlinear layers E_n,

$$E_{n+1}+E_{n-1}=[2\cos k-\alpha k\sin k(1+\lambda|E_n|^2)]E_n, \tag{4}$$

where $\alpha=\epsilon_0 a$ and $\lambda=\alpha_2/\alpha$. a is the thickness of the nonlinear layers. We have assumed that the linear medium is a vacuum ($\epsilon=1$), and the distance between the neighboring nonlinear layers, $d=a+b$, is taken as one unit length. The δ-function model can be viewed as an approximation when the nonlinear layer is thin compared with the effective wavelength within it.

In order to compare the results obtained for the δ-function model with the results obtained for the finite-width nonlinear layers, we have solved the wave equation [Eq. (1)] numerically for a system with nonlinear layers of width 0.1 unit length and linear layers ($\epsilon=1$) of width 0.9 unit length. The global transmission diagrams in the $k-E_t$ plane for the positive Kerr coefficient is shown in Fig. 1(b) as a gray scale plot, along with the results from the δ-function model [Fig. 1(a)]. The general features are remarkably similar. Good agreement is also obtained when the Kerr nonlinear coefficient is negative (Fig. 2). These transmission diagrams show features that have been understood through analysis of stable periodic orbits and various spectrum and stability bounds.[6]

A remarkable phenomenon occurred in the nonlinear response of superlattice structures is the existence of localized gap soliton solutions in the stop band of the linear regime.[4] In the previous study,[6] with the δ-function model, only in the negative Kerr media did we find gap soliton solutions. This is in disagreement with the conclusion of Chen and Mills that soliton solutions exist regardless of the sign of the nonlinearity.[4] In Fig. 3, we show the transmission diagram within the stop band for superlattices of finite widths with a positive nonlinear coefficient. Clearly, resonant trajectories exist. Examination of the solutions show well-localized waves symmetrically distributed at the center of the structure. Different resonance bands correspond to solutions containing different numbers of solitons, analogous to the situation with negative nonlinearity. Here the inadequacy of the δ-function model shows up, by not giving soliton solutions in the gap. For a negative nonlinear coefficient, the transmission diagram of the superlattice structure again is in good agreement with that of the δ-function model (see Fig. 4).

To understand why the δ-function model fails to describe the formation of a gap soliton in a superlattice with a positive nonlinear coefficient, we have to examine the physical

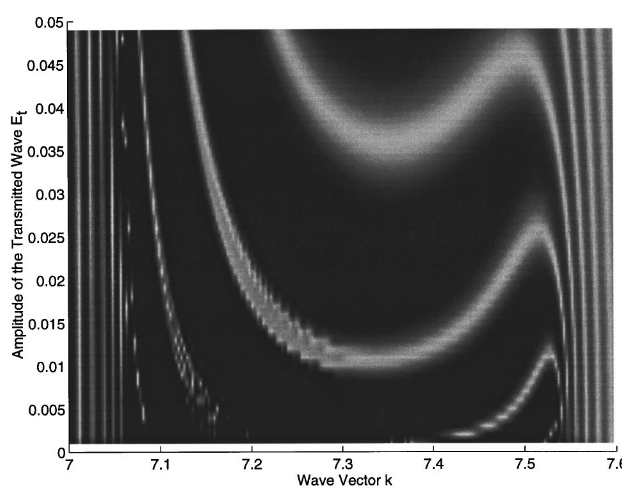

FIG. 3. The resonant transmission trajectories of single and multiple solitons in the third stop band of a nonlinear finite thickness superlattice of $L=80$ units with a positive Kerr coefficient. Linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=16$.

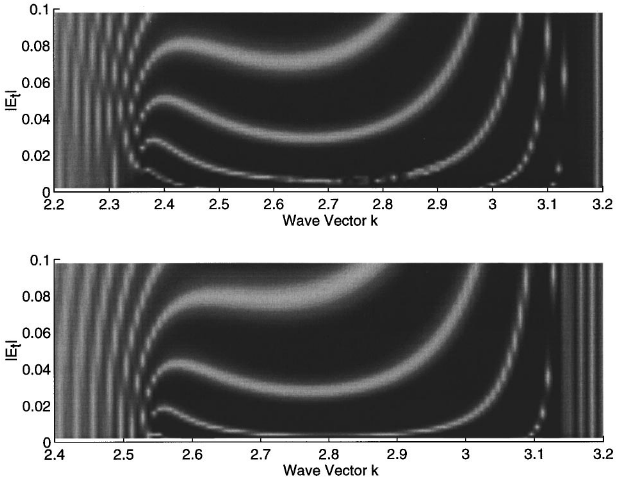

FIG. 4. The resonant transmission trajectories of single and multiple solitons in the first stop band of an $L=80$ multilayer system with a negative Kerr coefficient. (a) δ-function model with $\alpha=0.35$. (b) Finite-thickness bilayers with linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=3.5$.

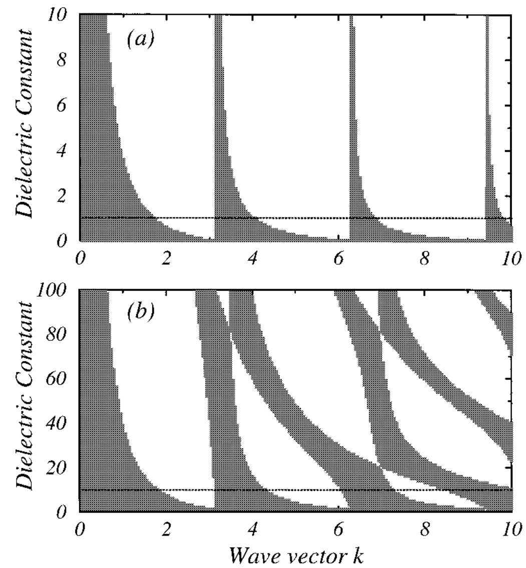

FIG. 6. The transmission band as a function of the effective dielectric constant at the nonlinear layers. $L=80$. (a) δ-function model with $\alpha=1$. (b) Linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=10$. The horizontal line indicates the bands in the linear regime.

mechanism in which gap solitons form when nonlinearity is incorporated into the model. This was elucidated with a mechanical analogy in the previous work.[6] The soliton forms only when the frequency is in the forbidden region of the spectrum in a linear system, i.e., in the gap. As the wave intensity varies along the structure, the dielectric constant of the nonlinear layers changes accordingly. Consequently the location of the effective transmission bands moves. For the soliton to form, the linearized transmission band has to shift in the right direction, such that the incident frequency merges into it. This is illustrated in Fig. 5, in which we show a single soliton profile [Fig. 5(a)] in the stop band. The corresponding effective transmission band edges are shown in Fig. 5(b), calculated from the linear dispersion relation using the local dielectric constant. Clearly, as the soliton intensity increases, the effective transmission band shifts toward the incident frequency and eventually takes it completely in the vicinity of the center of the soliton.

An examination of the effective transmission bands as a function of the effective dielectric constant in the nonlinear

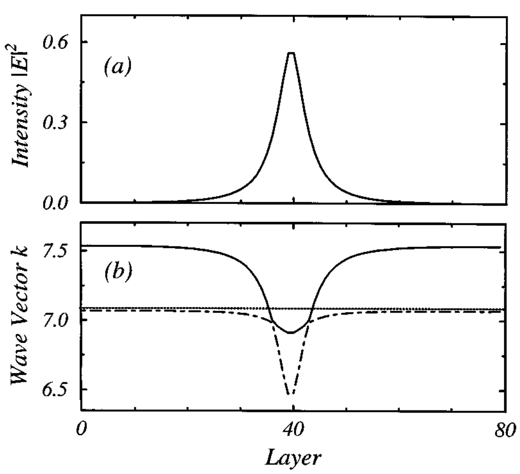

FIG. 5. Illustration of gap soliton formation in a system with $L=80$ layers with positive nonlinearity. Linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=16$. k=7.07369 and E_t=0.01. (a) Profile of a gap soliton, and (b) the local effective stop-band edges as determined from the soliton profile. The stop band extends from the dashed line (low-frequency stop-band edge) to the solid line (high-frequency stop-band edge). The thin line is the incident frequency. Clearly, the incident frequency merges inside the effective transmission band around the center of the soliton.

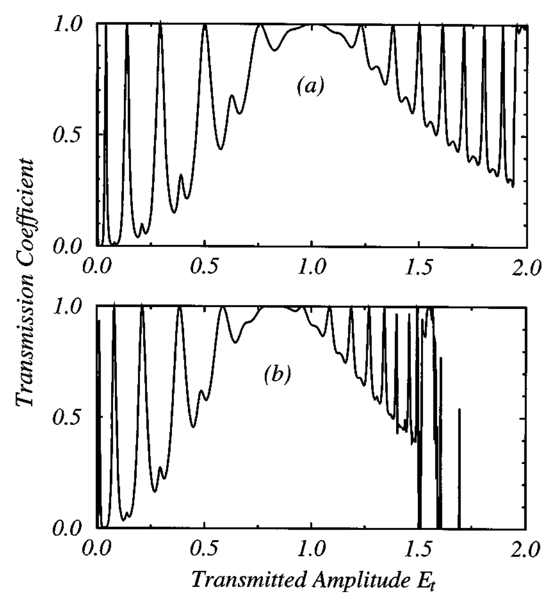

FIG. 7. Transmission coefficient as a function of the transmitted amplitude E_t, for an $L=80$ layer system with a negative nonlinearity. $k=3.0$. (a) δ-function model with $\alpha=0.35$. (b) Linear layers of thickness 0.9 and $\epsilon=1$, and nonlinear layers of thickness 0.1 and $\epsilon_0=3.5$.

layers shows clearly the difference between the δ-function model and the more realistic multilayer system (Fig. 6). For the negative nonlinearity (the portion below the dotted line), the collapse of the gap is described well with the δ function [Fig. 6(a)]. But for positive nonlinearity (the portion above the dotted line), the shift of the bottom of the bands toward lower frequency in the real system is completely missed in the δ function. This is not surprising, since the bottom of the band in the δ-function model is always located at $k=m\pi$. Thus the rigidness of the band edge in the δ-function model prevents the frequency below the bottom of the band from merging into the effective transmission band when the field intensity increases, and therefore hinders the formation of gap solitons.

In Fig. 7, we compare the transmission coefficient as a function of the transmitted amplitude obtained from the exact solution [Fig. 7(b)] and from the δ-function model [Fig. 7(a)], for a negative nonlinearity with a frequency below the bottom of the second band. Again, the qualitative features are the same. Notice the total transmission peaks at the resonance of gap soliton solutions.[8]

We have also investigated systems with thicker nonlinear layers, and found qualitatively similar behaviors. Qualitative agreement worsens, of course, as the nonlinear layer thickness increases. However, the essential features remain the same. Thus the δ-function model seems to be sufficiently adequate for a qualitative study of nonlinear response to radiation.

In conclusion, we examined many aspects of the nonlinear response in multilayer structures, and found the δ-function model quite adequate, aside from the obvious deficiency of processing a rigid bottom band edge. The model captures the most essential features in the transmission characteristics, and therefore should be widely used due to its simplicity.

Ames Laboratory is operated for the U. S. Department of Energy by Iowa State University under Contract No. W-7405-ENG-82. This work was supported by the Director of Energy Research, Office of Basic Energy Science, and Advanced Energy Projects. C.M.S. is grateful for the hospitality of the Research Center of Crete, FORTH, where this work was finished.

[1] H. M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic, Orlando, FL, 1985).

[2] H. M. Gibbs, S. L. McCall, and T. N. C. Venkatesant, Phys. Rev. Lett. **36**, 1135 (1976).

[3] H. G. Winful, J. H. Marburger, and E. Garmire, Appl. Phys. Lett. **35**, 379 (1979).

[4] Wei Chen and D. L. Mills, Phys. Rev. B **36**, 6269 (1987); Phys. Rev. Lett. **58**, 160 (1987).

[5] C. Martine de Sterke and J. E. Sipe, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1994), Vol. 33.

[6] Qiming Li, C. T. Chan, K. M. Ho, and C. M. Soukoulis, Phys. Rev. B **53**, 15 577 (1996)

[7] *Photonic Band Gaps and Localization*, edited by C. M. Soukoulis (Plenum, New York, 1993); *Photonic Band Gap Materials*, edited by C. M. Soukoulis (Klumer, Dordrecht, 1996).

[8] In Figs. 8 and 9 of Ref. 6, total transmission was not reached in some cases, due to an inappropriate boundary condition. This, however, does not affect any of the conclusions.