Economou and Soukoulis Respond: In our paper[1] we have proved the following two statements: (a) If the disordered linear chain is connected to an incoherent current source I, then the resistance of the chain defined as V_0/I (where V_0 is voltage across the chain) equals $(\pi\hbar/e^2)(1/|t|^2 - 1)$ in agreement with Landauer's result.[2] On the other hand, (b) if the disordered chain is connected to a perfect conducting ring, and a uniform voltage V_0 is applied across the chain only, the current at the chain is $I = V_0/R$ where $R = (\pi\hbar/e^2)(1/|t|^2)$.

In his Comment[3] Thouless claims that the quantity $R = (\pi\hbar/e^2)(1/|t|^2)$ appearing in case (b) above should not be taken as the resistance of the chain because its operational definition produces an oscillating current in the perfect conductor, and such oscillations violating the local charge neutrality of the system should not be allowed. We think that there is no compelling physical reason for accepting this opinion. It is true that the accumulated charges would produce an induced field, which, however could be canceled by an additional external field. This would not change R, since the conductance is the response to the total field (external plus induced) and not simply to the external field. On the other hand, we recognize that the oscillating current in the perfect conductor for the configuration (b) is an undesirable feature.

The second point made by Thouless is that if one uses a modified configuration (b′) including an additional uniform field V_1/L in the perfect conductor, such that the current uniformity is restored, then $R \equiv V_0/I$ is given again by $(\pi\hbar/e^2)(1/|t|^2 - 1)$ as in case (a). Such a result makes the case argued by Thouless very strong indeed.

By employing periodic boundary conditions we have calculated both the real and the imaginary parts of the conductivity, σ, of our system when connected to a perfect conducting ring. We found that the result for the case (b′) is as claimed by Thouless. Here we summarize some important results of our calculation.

The conductivity $\sigma(x,x')$ is given in general by the following equation (in units where $e = m = \hbar = 1$):

$$\sigma(x,x') = \frac{i}{\omega}\left[n\delta(x-x') - \sum_{\alpha\beta}\frac{W_{\alpha\beta}(x)W_{\alpha\beta}^*(x')}{\omega_{\beta\alpha}}\right] + \frac{1}{2}\sum_{\alpha\beta}\frac{W_{\alpha\beta}(x)W_{\alpha\beta}^*(x')}{i\omega_{\beta\alpha}}\left(\frac{1}{\omega_{\beta\alpha}-\omega-i\epsilon} - \frac{1}{\omega_{\beta\alpha}+\omega+i\epsilon}\right), \quad (1)$$

where $\omega_{\beta\alpha} \equiv \omega_\beta - \omega_\alpha$, $H\psi_\alpha = \omega_\alpha\psi_\alpha$, $H\psi_\beta = \omega_\beta\psi_\beta$, $\omega_\beta > \omega_F$, $\omega_\alpha < \omega_F$, ω is the frequency of the field,

$$W_{\alpha\beta}(x) = \psi_\alpha^*(x)\partial\psi_\beta(x)/\partial x - \psi_\beta(x)\partial\psi_\alpha^*(x)/\partial x,$$

n is the electron density, and $\epsilon \to 0+$.

We found that for $|x|, |x'| \ll L$, the term in brackets in Eq. (1) is zero and that

$$\sigma(x,x') = \frac{1}{\pi}\left[\exp\left\{i\frac{\omega}{v_F}|x-x'|\right\} - (1-|t|^2)\exp\left\{i\frac{\omega}{v_F}(|x|+|x'|)\right\}\right]. \quad (2)$$

This result was first obtained by Lee.[4] Equation (2) is not appropriate for obtaining the response to a uniform field extending over the whole perfect conductor (because of the restrictions $|x|, |x'| \ll L$.) One must return to the general Eq. (1), and perform the integration over x' and then the summations over α and β. By doing so, we found that the integral of the term in brackets in Eq. (1) is not zero but it is equal to $n|t|^2$. The other term in Eq. (1) gives a result proportional to $|r|^2 \equiv 1 - |t|^2$. So finally we obtain (for $|x| \ll L$)

$$\int_{-L/2}^{L/2}\sigma(x,x')dx' = \frac{i}{\omega}n|t|^2 - \frac{i}{\omega}n|r|^2\left\{\exp\left(i\frac{\omega}{v_F}|x|\right) - 1\right\} = \frac{i}{\omega}n - \frac{i}{\omega}n|r|^2\exp\left(i\frac{\omega}{v_F}|x|\right). \quad (3)$$

Taking into account Eqs. (2) and (3), and demanding that the current for $|x| \ll L$ must be uniform, we obtain for the configuration (b′) Thouless's result. It is worthwhile to point out that if the term in brackets in Eq. (1) were identically zero for all x, x' then the result for the resistance would be $1/|t|^2$ and not $1/|t|^2 - 1$.

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